Optimal Time-Consistent Monetary Policy and Sovereign Debt Currency Denomination

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Abstract

In the past two decades, the share of external sovereign debt in emerging economies denominated in foreign currency has fallen. I document that in Brazil, Colombia, Mexico and Peru, foreign currency debt was mainly substituted by debt denominated in local currency, while indexed debt represented a small proportion. In Uruguay, on the contrary, foreign currency debt was largely substituted by inflation-indexed debt. To study these differences, I use a small open economy model in which each period, a government that cannot default and lacks commitment optimally chooses inflation and the issuance of foreign currency, local currency and inflation-indexed debt. The choice of debt currency denomination balances hedging benefits and costs implied by each type of bond. On one hand, the exchange rate features a negative correlation with tradable endowment, making local currency and indexed debt useful securities in terms of hedging. On the other hand, the government is tempted to use inflation surprises (local currency debt) and real exchange rate depreciation surprises (local currency and indexed debt) to dilute the value of its debt. Foreign currency debt acts as a commitment device, because its value cannot be diluted. I derive analytical expressions to gain insight into the nature of these trade-offs and solve the model numerically to explain the difference in currency composition across countries. Large external debt-to-GDP ratios, long debt duration, and low inflation costs encourage more borrowing in inflation-indexed bonds and can explain the larger share of inflation-indexed debt in Uruguay compared to other Latin American countries.

JEL classification: E52, F34, H63

Keywords: Government Debt, Currency Denomination, Monetary Policy, Time Consistent Policy

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1 Introduction

Historically, external sovereign debt in emerging economies has largely been denominated in foreign currency, a phenomenon known in the international finance literature as the "original sin" (Eichengreen and Hausmann 1999). However, over the past two decades, the share of external sovereign debt denominated in foreign currency has evidenced a reduction (Arslanalp and Tsuda 2014). In this paper I address two questions. Has the debt denominated in foreign currency been replaced by debt denominated in local currency or indexed? What are the policy implications of these types of bonds and how does the government choose the currency denomination of its debt?

To answer the first question, I construct a dataset on foreign holdings of government debt issued internationally and domestically for five Latin American countries: Brazil, Colombia, Mexico, Peru and Uruguay. I follow Arslanalp and Tsuda (2014) and extend their work by distinguishing between debt denominated in local currency and indexed units. I find that in Brazil, Colombia, Mexico and Peru, external debt in foreign currency was mainly substituted by debt in local currency. In Uruguay, on the contrary, debt in foreign currency was largely substituted by inflation-indexed debt.

To answer the second question, I use a small open economy model in which inflation is costly and exchange rate is determined endogenously. Each period, a government that cannot default and lacks commitment regarding its monetary policy and bond issuance, optimally chooses inflation and the issuance of debt denominated in local currency, foreign currency and inflation-indexed units. I follow Ottonello and Perez (2019)'s analytical framework and extend it by introducing inflation-indexed debt to the government's portfolio choice.

Firstly, I analyze the trade-offs of each policy instrument by looking at the first order conditions of the problem. On one hand, local currency bonds and inflation-indexed bonds are useful tools in terms of hedging. As exchange rate (nominal and real) and tradable endowment are negatively correlated, the payoffs of these bonds decline in real terms when tradable endowment falls, helping to smooth tradable consumption. On the other hand, when unable to commit, the government has an ex-post incentive to generate surprise inflation and real exchange rate depreciation to dilute the real value of its local currency (both) and inflation-indexed bonds (only real exchange rate depreciation). Foreign investors anticipate these incentives, lowering the price for local currency and inflation-indexed debt. The government takes this into account, and chooses a lower amount of debt in local currency and inflation-indexed units than it will choose under commitment.

Secondly, I calibrate the model to the average Latin American country in the sample, Peru and Uruguay, and solve the model numerically. I find that large external debt-to-GDP ratios, long debt duration, and low inflation costs encourage more borrowing in inflation-indexed bonds and can explain the larger share of inflation-indexed debt in Uruguay compared to other Latin American countries.

In addition to Ottonello and Perez (2019), some recent papers examine the currency composition of sovereign debt, considering local and foreign currency bonds. S. Liu, Ma, and Shen (2021) embed the debt currency denomination in a sudden stop model, while Engel and Park (2022) analyze an optimal contract problem with outright default. These papers emphasize the hedging benefits and the inflationary bias of local currency bonds, but none of them incorporate inflation-indexed bonds as a policy tool. The innovation of this paper is to study empirically and theoretically the role of inflation-indexed bonds as an additional policy instrument for the government.

The paper proceeds as follows. The answer to the first (empirical) research question is provided in Section 2. To answer to the second (theoretical) research question, I first describe the model in Section 3, introduce the optimal policy problem in Section 4, and present the quantitative analysis in Section 5. Section 6 concludes.

2 Data

This section presents the main facts about the currency composition of external sovereign debt for five Latin American countries: Brazil, Colombia, Mexico, Peru and Uruguay. The countries were selected based on the size of the economy -they represent 70% of Latin America's gross domestic output- and data availability. The objective is to analyze how large debt positions in local currency and in inflation-indexed units are relative to foreign currency and how these relative positions have evolved over the last decades and across countries.

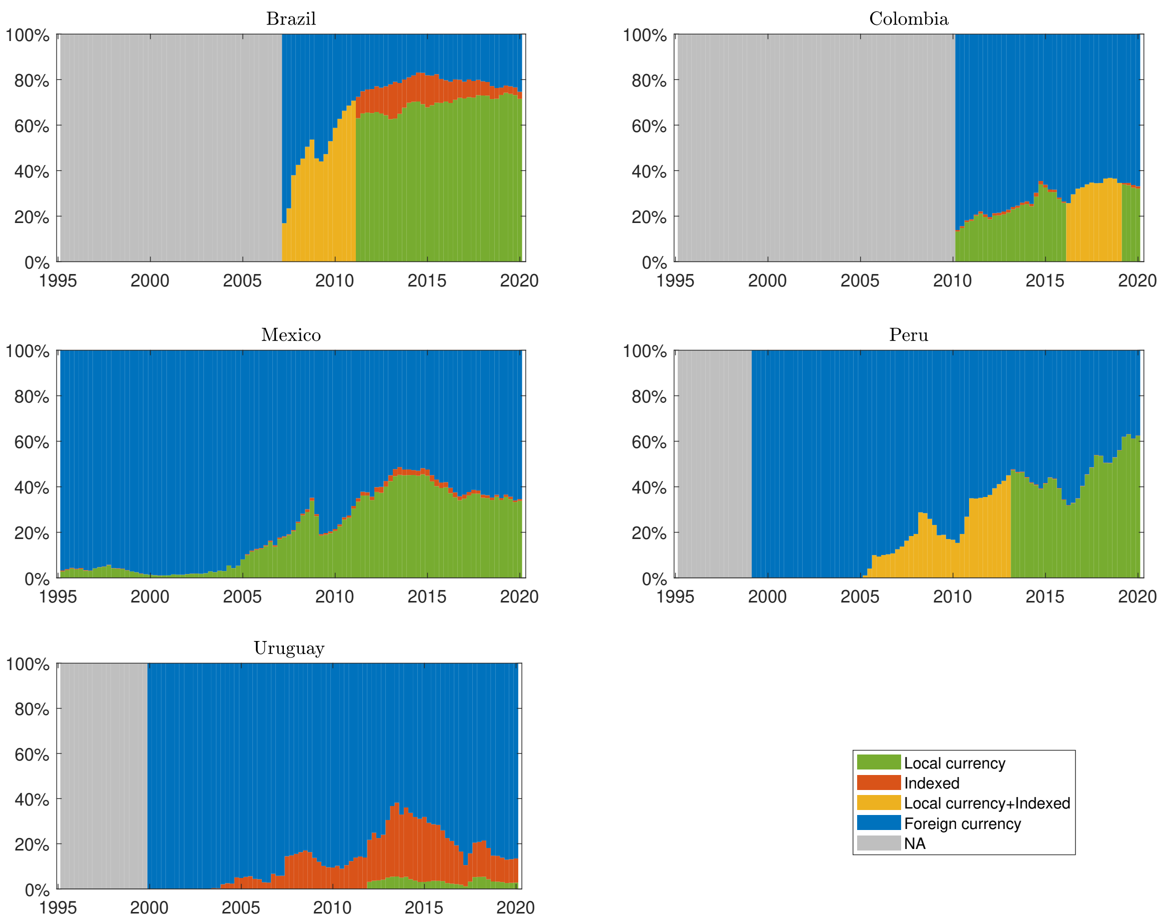

Figure 1: Currency composition of foreign holdings of sovereign debt.

I construct a dataset on foreign holdings of government debt issued internationally and domestically. For this purpose, I follow Arslanalp and Tsuda (2014) and extend their work by distinguishing between debt denominated in local currency and inflation-indexed units, which will be shown to have different characteristics and policy implications. In the case of Brazil, the data corresponds to Federal Public debt obtained from the monthly public debt reports, elaborated by the National Treasury. For Colombia, the data corresponds to Central Government debt obtained from the Ministry of Finance. In the case of Mexico, the data corresponds to Federal Public debt, obtained from the Ministry of Finance, for the internationally issued debt, and from the Central Bank of Mexico, for the domestically issued debt held by nonresidents. For Peru, the data corresponds to Non-Monetary Public Sector debt. The data that corresponds to internationally issued debt was obtained from the Central Bank of Peru, and the data for domestically issued debt from the Ministry of Finance. In the case of Uruguay, the data corresponds to Non-Monetary Public Sector debt, and it was obtained from the Central Bank of Uruguay. (See Appendix A for more details) The frequency and time-period of the data varies considerably across countries. For instance, the estimates for Mexico are constructed on a monthly basis starting in 1991, whereas the estimates for Colombia are quarterly and start in 2010.

Figure 1 depicts the dynamics of the currency composition of government debt held by foreigners in the five Latin American countries under study. Debt composition is tilted towards foreign currency, a well documented and studied characteristic of emerging economies. More than half of the foreign holdings of government debt are denominated in foreign currency, except for Brazil after 2010 and Peru in recent years. Moreover, in the last two decades there has been a reduction in the share of foreign currency debt, another fact more recently highlighted in the literature (see for example Ottonello and Perez 2019, S. Liu, Ma, and Shen 2021, and Engel and Park 2022). An innovation of this paper is to study the evolution of indexed debt. For the countries under study, this type of debt is indexed to the evolution of aggregate inflation, is issued domestically and its share is small compared to local currency debt. The exception is Uruguay, which most of its non-foreign currency denominated debt corresponds to inflation-indexed debt and some of it is issued internationally. Brazil also registered a significant share of inflation-indexed debt over a period of time. It reached 16% in 2013, but it fell to 3% by the end of the sample.

In the following sections, I build a model and derive analytical and numerical results to explain the differences across countries documented in this section.

3 Model

This section introduces a model to study optimal monetary policy and debt currency denomination. It closely follows Ottonello and Perez (2019), augmented to include inflation-indexed debt. A small open economy is populated by a continuum of identical risk-averse households that live forever, consume tradable and non-tradable goods, receive a stochastic endowment and lump-sum transfers net of taxes from the government, but cannot borrow or save. The government, who chooses inflation and debt issuance, is benevolent but lacks commitment and cannot default. The government can issue debt denominated in foreign currency, local currency, and indexed to inflation, which is priced by risk-neutral foreign investors that also have access to a risk-less bond denominated in foreign currency.

3.1 Households

In the small open economy, the preferences of the representative household are defined over an infinite stream of consumption and inflation:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - l(\pi_t)\right) \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, c_t denotes consumption, and $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, with P_t the aggregate price level. $u : R_+ \to R$ is a differentiable, increasing and concave function, and $l : R_+ \to R$ is a differentiable and convex function. The disutility from inflation captures the distortionary costs associated with this variable. In addition, the consumption good is a composite of tradable goods, $c_{T,t}$, and non-tradable goods, $c_{N,t}$

$$c_t = C(c_{T,t}, c_{N,t}) \tag{2}$$

where $C : R_+^2 \to R_+$ is a differentiable function, increasing in both arguments, concave, and homogeneous of degree one.

The representative household receives a stream of endowments of tradable goods, $y_{T,t}$, and non-tradable goods, $y_{N,t}$, and lump-sum transfers (net of taxes) from the government, T_t. Each period, the household faces the following budget constraint

$$P_{T,t}c_{T,t} + P_{N,t}c_{N,t} = P_{T,t}y_{T,t} + P_{N,t}y_{N,t} + T_t \tag{3}$$

where $P_{T,t}$ and $P_{N,t}$ are the prices of tradable and non-tradable goods, respectively, measured in local currency.

The representative household chooses state-contingent consumption plans, $\{c_t, c_{T,t}, c_{N,t}\}_{t=0}^{\infty}$, that maximize the lifetime utility (1), subject to the aggregation technology (2), the sequence of period budget constraints (3), and the given sequences of prices, $\{P_{T,t}, P_{N,t}\}_{t=0}^{\infty}$, endowments and government lump-sum transfers, $\{y_{T,t}, y_{N,t}, T_t\}_{t=0}^{\infty}$. The optimal choice between tradable and non-tradable goods is obtained by taking the ratio of the first order conditions (FOCs)

$$\frac{C_{c_{T,t}}}{C_{c_{N,t}}} = \frac{P_{T,t}}{P_{N,t}} \tag{4}$$

where $f_{x_i} = \partial f(x_1, ..., x_i, ..., x_n)/\partial x_i$.

3.2 Government

The government chooses inflation and external debt. There are three types of bonds available to the government: denominated in foreign currency, b_t^*, local currency, B_t, and indexed to inflation,

b_t^π. Each bond is sold at price Q_t^*, Q_t, and Q_t^π, respectively. Following Woodford (2001), I assume that the government bonds are actually a portfolio of many bonds that pay a declining coupon of δ^i local currency, inflation-indexed or foreign currency units, $j+1$ periods after they were issued, where $0 < \delta \leq \beta^{-1}$.[1] A measure of the duration of the bond is given by $(1-\beta\delta)^{-1}$, which is used to calibrate δ to capture the observed maturity structure of government debt. Note that when $\delta = 0$, the bonds are reduced to one period bonds.

The government budget constraint expressed in local currency is then given by

$$e_t Q_t^* b_t^* + P_t \left(Q_t b_t + Q_t^\pi b_t^\pi \right) = e_t (1 + \delta Q_t^*) b_{t-1}^* + P_t \left[(1 + \delta Q_t) \frac{b_{t-1}}{\pi_t} + (1 + \delta Q_t^\pi) b_{t-1}^\pi \right] + T_t \tag{5}$$

where $Q_t b_t = Q_t B_t / P_t$ is real value of local currency bonds, and e_t is the nominal exchange rate, i.e. the price of foreign currency in terms of local currency.

3.3 Rest of the World

The rest of the world is populated by a continuum of identical risk-neutral households that have access to the bonds issued by the small open economy's government and a risk-free bond denominated in foreign currency that pays gross international rate R. Bond prices are then given by

$$Q_t^* = \frac{1}{R - \delta} \tag{6}$$

$$Q_t = \frac{1}{R} E_t \left[\frac{e_t}{e_{t+1}} (1 + \delta Q_{t+1}) \right] \tag{7}$$

$$Q_t^\pi = \frac{1}{R} E_t \left[\frac{e_t}{e_{t+1}} \frac{P_{t+1}}{P_t} (1 + \delta Q_{t+1}^\pi) \right] = \frac{1}{R} E_t \left[\frac{r_t}{r_{t+1}} (1 + \delta Q_{t+1}^\pi) \right] \tag{8}$$

where $r_t = e_t / P_t$ is the real exchange rate (RER).

3.4 Equilibrium

In equilibrium, the market for non-tradable goods clears

$$c_{N,t} = y_{N,t} \tag{9}$$

Assuming the law of one price holds for tradable goods ($P_{T,t} = e_t P_{T,t}^*$) and normalizing the international price of tradable goods to one, we get that $P_{T,t} = e_t$. Given the assumed preferences (1) and the aggregation technology (2), the aggregate price level of the small open economy is given by $P_t = e_t \left(C_{c_T} \left(\frac{c_{T,1}}{y_{N,t}}, 1 \right) \right)^{-1}$.[2] For later use, we define the inverse of the equilibrium nominal exchange

[1] By issuing one bond in local currency (inflation-indexed or foreign currency) in period t, the government promises to repay one unit of local currency (inflation-indexed or foreign currency) in period $t+1$, δ in period $t+2$, δ^2 in period $t+3$, and so on, and in exchange receives Q_t (Q_t^π or Q_t^*) units of local currency (inflation-indexed or foreign currency) in period t.

[2] See Uribe and Schmitt-Grohe (2017).

rate, n, as

$$e_t^{-1} = n\left(P_t, \frac{c_{T,t}}{y_{N,t}}\right), \qquad n\left(P_t, \frac{c_{T,t}}{y_{N,t}}\right) = \frac{1}{P_t}\left(C_{c_T}\left(\frac{c_{T,t}}{y_{N,t}}, 1\right)\right)^{-1} \tag{10}$$

where $n\left(P_t, \frac{c_{T,t}}{y_{N,t}}\right)$ is a differentiable function, decreasing in its first argument and increasing in its second argument. Note that

$$\left(\frac{e_t}{P_{t-1}}\right)^{-1} = n\left(\pi_t, \frac{c_{T,t}}{y_{N,t}}\right), \qquad r_t^{-1} = n\left(1, \frac{c_{T,t}}{y_{N,t}}\right)$$

Aggregating in the small open economy households and government budget constraints, (3) and (5), imposing the market clearing condition in the non-tradable sector (9), and using the definition of the inverse of the equilibrium nominal rate (10), we obtain the resource constraint of the small open economy, expressed in units of tradable consumption

$$\begin{aligned} c_{T,t} = {} & y_{T,t} - b^*_{t-1} - n\left(\pi_t, \frac{c_{t,T}}{y_{N,t}}\right) b_{t-1} - n\left(1, \frac{c_{t,T}}{y_{N,t}}\right) b^\pi_{t-1} \\ & + Q^*_t\left(b^*_t - \delta b^*_{t-1}\right) + n\left(1, \frac{c_{t,T}}{y_{N,t}}\right)\left[Q_t\left(b_t - \delta\frac{b_{t-1}}{\pi_t}\right) + Q^\pi_t\left(b^\pi_t - \delta b^\pi_{t-1}\right)\right] \end{aligned} \tag{11}$$

Given initial levels of government debt b^*_{-1}, b_{-1}, and b^π_{-1}, a state-contingent sequence of endowments $\{y_{T,t}, y_{N,t}\}_{t=0}^\infty$, and government policies $\{b^*_t, b_t, b^\pi_t\}_{t=0}^\infty$, a competitive equilibrium is a state-contingent sequence of private sector allocations $\{c_{T,t}, c_{N,t}\}_{t=0}^\infty$ and prices $\{Q^*_t, Q_t, Q^\pi_t, e_t, \pi_t\}_{t=0}^\infty$, such that: private allocations solve the household's problem given equilibrium prices, transfers (net of taxes) satisfy the government budget constraint (5), debt prices satisfy (6), (7) and (8), and market for non-tradable goods clears (9).

4 Optimal Policy

This section outlines the policy problem of the government, which has to choose a set of government policies $\{\pi_t, b^*_t, b_t, b^\pi_t\}_{t=0}^\infty$ in order to maximize the utility of the representative household (1), subject to the constraints implied by the competitive equilibrium defined in the previous section. Before introducing the time-consistent problem, which is the focus of this analysis, I briefly lay out the problem under commitment to use as a benchmark. We focus on the notion of a Markov perfect equilibrium in which policies depend on payoff-relevant states. In other words, the solution to the system of equations that defines the policy problem will be a set of time-invariant Markov-perfect equilibrium policy rules mapping the vector of states to the optimal decisions for each variable and for all moment in time. Regarding the sequence of endowments, I assume that y_N is constant over time while y_T follows a Markov process with transition probability $g_y(y_T, y'_T)$.

4.1 Commitment and Discretion

4.1.1 Commitment

When the government can commit to future policies, the policy problem is given by choosing $\{c_{T,t}, \pi_t, b_t^*, b_t, b_t^\pi\}$ in order to maximize household's lifetime utility (1), subject to the resource constraint (11) and bond prices (6, 7, 8). By committing to an entire path of policy instruments, the government is able to influence expectations in order to improve the policy trade-offs they face. The Lagrangian and the resultant set of FOCs are derived in Appendix A.

The commitment equilibrium is determined by the system given by the FOCs (32) in Appendix B.1.1, the constraints (11, 6, 7, 8) and the exogenous process for the endowment of tradable and nontradable goods.

4.1.2 Discretion

When the government lacks commitment, it seeks to maximize the value function:

$$V(s_t) = \max_{c_{T,t},\pi_t,b_t^*,b_t,b_t^\pi} u\left(C\left(c_{T,t}, y_N\right)\right) - l(\pi_t) + \beta E_t\left[V(s_{t+1})\right] \tag{12}$$

subject to the resource constraint (11) and bond prices (6, 7, 8). Even though the government chooses the same variables than under commitment, it cannot make time-inconsistent promises about their future behavior to have a beneficial impact on current policy trade-offs through expectations. Instead, foreign lenders anticipate the incentives faced by the government each period and form expectations accordingly. However, the government can still influence those expectations by affecting the states the next period's government inherits. To capture this, future expectations are replaced by the following state-dependent auxiliary functions:

$$\mathcal{X}(s_{t+1}) = n\left(\pi_{t+1}, \frac{c_{T,t+1}}{y_N}\right)(1 + \delta Q_{t+1}) \tag{13}$$

$$\mathcal{Z}(s_{t+1}) = n\left(1, \frac{c_{T,t+1}}{y_N}\right)(1 + \delta Q^\pi_{t+1}) \tag{14}$$

in the local currency and indexed bond pricing equations, respectively. These auxiliary functions reflect the fact that, in equilibrium, we can map exogenous variables to the steady state and expectations are formed rationally based on that mapping. The current government takes this into account when setting its policy. The price of local currency and inflation-indexed debt are determined by the following recursive equations

$$Q(s_t) = \frac{1}{n\left(1, \frac{c_{T,t}}{y_N}\right)} \frac{1}{R} E_t[\mathcal{X}(s_{t+1})] \tag{15}$$

$$Q^\pi(s_t) = \frac{1}{n\left(1, \frac{c_{T,t}}{y_N}\right)} \frac{1}{R} E_t[\mathcal{Z}(s_{t+1})] \tag{16}$$

The discretionary equilibrium is determined by the system given by the FOCs (34) in Appendix B.1.2; the resource constraint (11); auxiliary equations (13) and (14); bond prices (15) and (16); and the exogenous process for the endowment of tradable goods. Using the notion of Markov perfect equilibrium, the solution to this system is a set of time-invariant policy rules $x_t = H(s_t)$ mapping the vector of states $s_t = \{b^*_{t-1}, b_{t-1}, b^\pi_{t-1}, y_{T,t}\}$ to the optimal decisions for $x_t = \{c_{T,t}, \pi_t, b^*_t, b_t, b^\pi_t, Q^*_t, Q_t, Q^\pi_t\}$ for all $t \geq 0$.

4.2 Policy Trade-Offs

The policy trade-offs faced by the government can be characterized by analyzing the FOCs.

4.2.1 Dilution through real exchange rate

First, consider the FOC for tradable consumption

$$
\begin{aligned}
& u'(c_t)C_{c_T,t} - \lambda_t \underbrace{\left[1 + n_c\left(\pi_t, \frac{c_{T,t}}{y_N}\right) b_{t-1} + n_c\left(1, \frac{c_{T,t}}{y_N}\right) b^\pi_{t-1}\right]}_{\text{Dilution through RER}} \\
& \underbrace{-\lambda_t \left\{1 - n_c\left(1, \frac{c_{T,t}}{y_N}\right)\left[Q_t\left(b_t - \delta\frac{b_{t-1}}{\pi_t}\right) + Q^\pi_t\left(b^\pi_t - \delta b^\pi_{t-1}\right)\right]\right\}}_{\text{Only under commitment}} \\
& \underbrace{+\mu_t \frac{1}{R}\frac{n_c\left(\pi_t, \frac{c_{T,t}}{y_N}\right)}{n\left(1, \frac{c_{T,t}}{y_N}\right)} Q_t + \mu^\pi_t \frac{1}{R}\frac{n_c\left(1, \frac{c_{T,t}}{y_N}\right)}{n\left(1, \frac{c_{T,t}}{y_N}\right)} Q^\pi_t}_{\text{Only under commitment}} \\
& \underbrace{-\frac{1}{\beta}\frac{1}{R}\frac{1}{n\left(1, \frac{c_{T,t-1}}{y_N}\right)}\left[\mu_{t-1} n_c\left(\pi_t, \frac{c_{T,t}}{y_N}\right)(1 + \delta Q_t) + \mu^\pi_{t-1} n_c\left(1, \frac{c_{T,t}}{y_N}\right)(1 + \delta Q^\pi_t)\right]}_{\text{Only under commitment}} = 0 \qquad (17)
\end{aligned}
$$

Lines two to four in expression (17) are only present under commitment and reflect the fact that the government can directly affect expectations and therefore bond prices. The first line is present both under commitment and discretion. Under discretion and with only foreign currency bonds, the marginal benefit of raising one additional unit of good with debt is $u'(c_t)C_{c_T,t}$. But in the presence of local currency or inflation-indexed bonds, increasing one additional unit of good with debt increases tradable consumption by less than one, i.e. the denominator in (17) is greater than one. This is because higher tradable consumption appreciates the real exchange rate ($n_c(1, c_{T,t}/y_N) > 0$), which increases the value of local currency or indexed debt repayment and decreases tradable consumption. This creates an incentive for the government to depreciate the real exchange rate ex-post to dilute the real value of debt repayment. To do it, tradable consumption has to be postponed, result that can be attained by issuing less debt at the cost of distorting intertemporal consumption decisions.

4.2.2 Dilution through inflation

Second, consider the FOC for inflation, which highlights the nature of the inflationary bias contained in the model,

$$-l'(\pi_t) = \underbrace{\lambda_t \, n_\pi \left(\pi_t, \frac{c_{T,t}}{y_N}\right)(1+\delta Q_t) b_{t-1}}_{\text{Dilution through inflation}} + \underbrace{\mu_{t-1}\frac{1}{\beta R} n_\pi \left(\pi_t, \frac{c_{T,t}}{y_N}\right)(1+\delta Q_t) r_{t-1}}_{\text{Only under commitment}} \tag{18}$$

The left-hand-side measures the cost of raising inflation. The terms on the right-hand-side of (18) measure the benefits of raising inflation, and are only present when there is a non-zero stock of debt in local currency. The first term captures the fact that higher inflation dilutes the value of outstanding debt in local currency measured in units of tradable consumption and enables a higher consumption by saving resources. This benefit is higher, the higher the stock and the longer the maturity of debt in local currency. The second term, only present under commitment, captures the government's promise not to use inflation to reduce bond prices. Then, under discretion, ex-post the government has an incentive to increase inflation.

4.2.3 Hedging benefits and costs derived from the lack of commitment

Third, consider the FOC for each type of bond:

$$[b_t^*]: \quad \underbrace{\beta E_t\left[\lambda_{t+1}\right]\left(1+\frac{\delta}{R-\delta}\right) - \lambda_t \frac{1}{R-\delta}}_{\text{Consumption smoothing}}$$

$$\underbrace{-\lambda_t \left\{\frac{1}{R} E_t[\mathcal{X}_{b^*}(s_{t+1})]\left(b_t - \delta\frac{b_{t-1}}{\pi}\right) + \frac{1}{R} E_t[\mathcal{Z}_{b^*}(s_{t+1})]\left(b_t^\pi - \delta b_{t-1}^\pi\right)\right\}}_{\text{Only under discretion}} = 0 \tag{19}$$

$$[b_t]: \quad \underbrace{\beta E_t\left[\lambda_{t+1}\frac{1}{r_{t+1}}\frac{1}{\pi_{t+1}}(1+\delta Q_{t+1})\right] - \lambda_t \frac{1}{r_t} Q_t}_{\text{Consumption smoothing}}$$

$$\underbrace{-\lambda_t \frac{1}{R}\left\{E[\mathcal{X}_b(s_{t+1})]\left(b_t - \delta\frac{b_{t-1}}{\pi_t}\right) + E_t[\mathcal{Z}_b(s_{t+1})]\left(b_t^\pi - \delta b_{t-1}^\pi\right)\right\}}_{\text{Only under discretion}} = 0 \tag{20}$$

$$[b_t^\pi]: \quad \underbrace{\beta E_t\left[\lambda_{t+1}\frac{1}{r_{t+1}}\left(1+\delta Q_{t+1}^\pi\right)\right] - \lambda_t \frac{1}{r_t} Q_t^\pi}_{\text{Consumption smoothing}}$$

$$\underbrace{-\lambda_t \frac{1}{R}\left\{E_t[\mathcal{X}_{b^\pi}(s_{t+1})]\left(b_t - \delta\frac{b_{t-1}}{\pi_t}\right) + E_t[\mathcal{Z}_{b^\pi}(s_{t+1})]\left(b_t^\pi - \delta b_{t-1}^\pi\right)\right\}}_{\text{Only under discretion}} = 0 \tag{21}$$

Equations (19), (20) and (21) describe the government's optimal debt policy regarding foreign currency, local currency and inflation-indexed bonds, respectively.

- **Hedging benefits**

The first line in each of these equations gives the optimal trade-off between current and future distortions associated with the need to satisfy the government's intertemporal budget constraint - consumption smoothing (hand-to-mouth households). The government trades-off the short-run costs of reducing the stock of debt against the discounted value of the long-term benefits of lower debt. These terms are present under both commitment and discretion.

To study this effect in isolation, consider a case where inflation is costless ($l(\pi_t) = 0$), the real exchange rate is independent from consumption ($n_c \left(1, \frac{c_{T,t}}{y_N}\right) = 0$) and bonds are only one-period ($\delta = 0$). Then,

$$u'(c_t)C_{c_T,t} = \beta R E_t \left[u'(c_{t+1})C_{c_T,t+1}\right] \tag{22}$$

$$COV\left[u'(c_t)C_{c_T,t}, (e_t/P_{t-1})^{-1}\right] = 0 \quad \text{with local currency bonds} \tag{23}$$

$$COV\left[u'(c_t)C_{c_T,t}, r_t^{-1}\right] = 0 \quad \text{with inflation-indexed bonds} \tag{24}$$

Equation (22) defines the optimal path for tradable consumption. Equations (23) and (24) arise from the possibility of issuing bonds in local currency and inflation-indexed, respectively. According to these equations, the marginal utility of tradable consumption is isolated from fluctuations in the nominal exchange rate, in the case of being able to issue local currency bonds, and in the real exchange rate, in the case of inflation-indexed bonds. When nominal exchange rate is perfectly negatively (positively) correlated with tradable income, the government can obtain perfect tradable consumption smoothing by issuing positive (negative) debt in local currency. [3] Similarly, when real exchange rate is perfectly negatively (positively) correlated with tradable income, the government can obtain perfect tradable consumption smoothing by issuing positive (negative) inflation-indexed debt. A well-known stylized fact in emerging economies, documented for our sample of Latin American countries in Table 3, is that exchange rates are negatively correlated with aggregate tradable income. Then, local currency and inflation-indexed bonds can be a useful hedging against endowment risk.

- **Discipline effects**

The second line in equations (19), (20) and (21) captures wedges that are introduced when the government lacks commitment. Under discretion, these expressions are generalized Euler equations, which include partial derivatives of policy functions with respect to debt due to time-consistency requirements. These partial derivatives capture the effect of higher debt on bond prices. In general, the form of these auxiliary functions, $\mathcal{X}(s_t)$ and $\mathcal{Z}(s_t)$, is unknown, which is why we need to use numerical methods to solve the policy problem.

If the government cannot keep its promises, foreign lenders anticipate that higher debt in local currency increases the government's desire to introduce inflation and real exchange rate depreciation surprises, and demand a higher return on local currency bonds. Similarly, foreign lenders anticipate

[3] See Ottonello and Perez (2019) and Korinek (2009).

that higher inflation-indexed debt increases government's temptation to generate real exchange rate depreciation surprises, and demand a higher return on inflation-indexed bonds.

I focus on one-period bonds ($\delta = 0$) and assume that the endowment of tradables is constant to analytically explore how the government internalizes the lower prices on local currency and indexed bonds. The generalized Euler equations can be expressed in this case as (see derivation in Appendix B.2):

$$[b_t^*] : u_t' C_{c_T,t} = \beta R u_{t+1}' C_{c_T,t+1} \underbrace{\left(1 + n_{c,t} b_{t-1} + m_{c,t} b_{t-1}^{\pi}\right)}_{\text{Dilution through RER}} \frac{1}{\underbrace{1 + \underbrace{b_t n_{\pi,t+1} \pi_{\boldsymbol{b}^*,t+1}}_{\substack{\text{due to dilution} \\ \text{through inflation}}} + \underbrace{\left(n_{c,t+1} b_t + m_{c,t+1} b_t^{\pi}\right) \frac{1}{R} \frac{\partial \left(n_{t+2} b_{t+1} + m_{t+2} b_{t+1}^{\pi} + b_{t+1}^*\right)}{\partial \boldsymbol{b}_t^*}}_{\substack{\text{due to dilution} \\ \text{through RER}}}}_{\text{Discipline effect}}} \tag{25}$$

$$[b_t] : u_t' C_{c_T,t} = \beta R u_{t+1}' C_{c_T,t+1} \underbrace{\left(1 + n_{c,t} b_{t-1} + m_{c,t} b_{t-1}^{\pi}\right)}_{\text{Dilution through RER}} \frac{1}{\underbrace{1 + \underbrace{\frac{b_t n_{\pi,t+1} \pi_{\boldsymbol{b},t+1}}{n_{t+1}}}_{\substack{\text{due to dilution} \\ \text{through inflation}}} + \underbrace{\frac{\left(n_{c,t+1} b_t + m_{c,t+1} b_t^{\pi}\right) \frac{1}{R} \frac{\partial \left(n_{t+2} b_{t+1} + v_{t+2} b_{t+1}^{\pi} + b_{t+1}^*\right)}{\partial \boldsymbol{b}_t}}{n_{t+1}}}_{\substack{\text{due to dilution} \\ \text{through RER}}}}_{\text{Discipline effect}}} \tag{26}$$

$$[b_t^{\pi}] : u_t' C_{c_T,t} = \beta R u_{t+1}' C_{c_T,t+1} \underbrace{\left(1 + n_{c,t} b_{t-1} + m_{c,t} b_{t-1}^{\pi}\right)}_{\text{Dilution through RER}} \frac{1}{\underbrace{1 + \underbrace{\frac{b_t n_{\pi,t+1} \pi_{\boldsymbol{b}^{\pi},t+1}}{m_{t+1}}}_{\substack{\text{due to dilution} \\ \text{through inflation}}} + \underbrace{\frac{\left(n_{c,t+1} b_t + m_{c,t+1} b_t^{\pi}\right) \frac{1}{R} \frac{\partial \left(n_{t+2} b_{t+1} + v_{t+2} b_{t+1}^{\pi} + b_{t+1}^*\right)}{\partial \boldsymbol{b}_t^{\pi}}}{m_{t+1}}}_{\substack{\text{due to dilution} \\ \text{through RER}}}}_{\text{Discipline effect}}} \tag{27}$$

where $n_t = (\pi_t, c_{T,t})$, $m_t = (1, c_{T,t})$, $n_{\pi,t} = n_\pi(\pi_t, c_{T,t})$, $n_{c,t} = n_c(\pi_t, c_{T,t})$ and $m_{c,t} = m_c(1, c_{T,t})$. The term that captures the dilution through real exchange rate is common to (25), (26), and (27). This concept was explained above when analyzing the FOC for tradable consumption, and refers to the fact that when rising one additional unit of good with debt, consumption of tradables increases by $1/\left(1 + n_{c,t} b_{t-1} + m_{c,t} b_{t-1}^{\pi}\right)$. The term that captures the discipline effect reflects the higher returns demanded by the foreign lenders as a consequence of the ex-post incentives for the government to dilute the real value of local currency and inflation-indexed debt. When the government issues bonds in local currency, it is tempted to inflate away its real value by generating surprise inflation. The term that captures the discipline effect related to the dilution through inflation depends on the reduction in debt repayment in local currency, $n_{\pi,t+1}b_t$ (common for all types of bonds), and on the sensitivity of the optimal inflation to the type of bond considered, $\pi_{b^*,t+1}$, $\pi_{b,t+1}$, $\pi_{b^\pi,t+1}$. Moreover, when the government issues bonds in local currency or indexed to inflation, it is tempted to dilute their real value by depreciating the real exchange rate. The term that captures the discipline effect related to the dilution through real exchange rate depends on the reduction in debt repayment in local currency and inflation-indexed, $n_{c,t+1}b_t + m_{c,t+1}b_t^\pi$ (common for all types of bonds), and on the sensitivity of future debt choices to the current choice of bond considered, $\frac{\partial\left(n_{t+2}b_{t+1}+v_{t+2}b^\pi_{t+1}+b^*_{t+1}\right)}{\partial \boldsymbol{b}^*_t}$, $\frac{\partial\left(n_{t+2}b_{t+1}+v_{t+2}b^\pi_{t+1}+b^*_{t+1}\right)}{\partial \boldsymbol{b}_t}$, $\frac{\partial\left(n_{t+2}b_{t+1}+v_{t+2}b^\pi_{t+1}+b^*_{t+1}\right)}{\partial \boldsymbol{b}^\pi_t}$.

To summarize, from analyzing the FOCs for consumption and inflation we conclude that, under discretion, there is a real exchange rate depreciation and an inflation bias. With local currency and indexed debt, the government has an incentive, ex-post, to depreciate the real exchange rate to dilute the real value of these type of bonds. Similarly, with local currency debt, the government has an incentive to increase inflation to reduce the real value of this type of debt. Looking at the FOCs for each type of bond, we first observe that local currency and inflation-indexed bonds are useful instruments to smooth tradable consumption. As nominal and real exchange rate are negatively correlated with tradable endowment, the real value of their payoffs go up in “bad times”, providing hedging against tradable endowment risk. Finally, we note that foreign investors anticipate the ex-post incentives of the government to generate surprise inflation and real exchange rate depreciation, lowering the price for local currency and inflation-indexed debt. The government takes this into account, and chooses a lower amount of debt in local currency and indexed units than it will choose under commitment.

5 Quantitative Analysis

In this section I calibrate the model to match the average Latin American country in the sample and two particular countries: Peru and Uruguay. I use the quantified model to offer potential explanations for the differences between countries.

5.1 Calibration

Before moving to the calibration of the model, we need to make some assumptions. One period corresponds to one year. The period utility function takes the following form

$$u(c_{T,t}, c_{N,t}) - l(\pi_t) = \frac{(c^\alpha_{T,t}c^{1-\alpha}_{N,t})^{1-\sigma}}{1-\sigma} - \frac{\psi_\pi}{2}(\pi_t - \bar{\pi})^2 \tag{28}$$

where α is the share of tradables in aggregate consumption, σ corresponds to the coefficient of risk aversion, ψ_π is the parameter that governs the disutility of excess inflation, and $\bar{\pi}$ is some benchmark inflation rate.

Table 1: Calibration for the average Latin American country

Description	Parameter	Value	Source/Target
From literature:			
Risk aversion	σ	5	
Tradable share in utility	α	0.5	Ottonello and Perez (2019)
Cost of inflation	ψ_π	7.08	
Risk free interest rate	R	1.04	
Nontradable endowment	y_N	1	Normalization
Calibrated to fit targets:			
Discount factor	β	0.960	Average external debt = 17%
Benchmark inflation	$\bar{\pi}$	1.043	Average inflation = 1.049
Decay rate of bonds	δ	0.89	Average debt duration = 7.0
Autocorrelation of y_T	ρ_{y_T}	0.53	Estimation, data tradable output
Standard deviation of y_T	σ_{y_T}	0.03	Estimation, data tradable output
Cost of issuing bonds	ψ_b	0.065	Same across types of bonds
Target of FC bonds	$\bar{B}^*$	0.22	Average share of FC bonds = 67%
Target of LC bonds	$\bar{B}$	0.10	Average share of LC bonds = 27%
Target of indexed bonds	$\bar{B}^\pi$	0.01	Average share indexed bonds = 6%

The process for the tradable endowment is assumed to follow an AR(1) process in logs

$$\log\, y_{T,t} = \rho_{y_T} \log\, y_{T,t-1} + \epsilon_t, \tag{29}$$

where $\epsilon_t \sim N\left(0, \sigma_{y_T}^2\right)$.

Additionally, I introduce a quadratic cost for the issuance of bonds, which is meant to capture the distortions not discussed in this paper. The introduction of foreign currency bond issuance costs is extensively used in small open economies, see Schmitt-Grohe and Uribe (2003). S. Liu, Ma, and Shen (2021) incorporate local currency bond issuance costs. I am extending this approach to inflation-indexed bonds. Then, the resource constraint becomes:

$$\begin{aligned} c_{T,t} = {} & y_{T,t} - b_{t-1}^* - \frac{c_{T,t}^{1-\alpha}}{\pi_t} \frac{b_{t-1}}{\alpha} - c_{T,t}^{1-\alpha} \frac{b_{t-1}^\pi}{\alpha} \\ & + \frac{1}{R-\delta}(b_t^* - \delta b_{t-1}^*) + \frac{1}{R} E\left[\mathcal{X}(s_{t+1})\right] \frac{1}{\alpha}\left(b_t - \delta \frac{b_{t-1}}{\pi_t}\right) + \frac{1}{R} E\left[\mathcal{Z}(s_{t+1})\right] \frac{1}{\alpha}\left(b_t^\pi - \delta b_{t-1}^\pi\right) \\ & - \frac{\psi_b}{2}\left(\frac{1}{R-\delta} b_t^* - \bar{B}^*\right)^2 - \frac{\psi_b}{2}\left(\frac{1}{R} E\left[\mathcal{X}(s_{t+1})\right] \frac{b_t}{\alpha} - \bar{B}\right)^2 - \frac{\psi_b}{2}\left(\frac{1}{R} E\left[\mathcal{Z}(s_{t+1})\right] \frac{b_t^\pi}{\alpha} - \bar{B}^\pi\right)^2 \end{aligned} \tag{30}$$

where $s_t = \left(\frac{b_{t-1}^*}{\alpha}, \frac{b_{t-1}}{\alpha}, \frac{b_{t-1}^\pi}{\alpha}, y_{T,t}\right)$. The FOCs of the calibrated model are described in Appendix B.3.

The calibrated parameter values are summarized in Table 1. The risk aversion coefficient, σ, the share of tradables in the consumption aggregator, α, the international risk-free interest rate, R, and the cost of inflation, ψ_π are taken from Ottonello and Perez (2019). These parameters are used to calibrate the model for the average Latin American country as well as for Peru and Uruguay. Following their work, the risk aversion coefficient is set to 5, which is within the upper

Table 2: Calibration for Peru and Uruguay

Description	Parameter	Value		Target
		Peru	Uruguay	
Calibrated to fit targets:				
Discount factor	β	0.961	0.953	Average external debt = 14%, 31%
Target of inflation	$\bar{\pi}$	1.023	1.063	Average inflation = 1.028, 1.076
Decay rate of bonds	δ	0.90	0.95	Average debt duration = 7.5, 10.6
Autocorrelation of y_T	ρ_{y_T}	0.40	0.71	Estimation, data tradable output
Standard deviation of y_T	σ_{y_T}	0.02	0.03	Estimation, data tradable output

values considered in the macroeconomic literature and in the lower values considered in the finance literature. The share of tradables in the consumption aggregator is set to 0.5, which is similar to the value used by S. Liu, Ma, and Shen (2021) (0.4). The international risk-free interest rate, R, is set to 1.04, a standard value in the literature. The cost of inflation is set to 7.08. The level of nontradable endowment, y_N, is normalized to one.

The remaining parameters are calibrated to match moments in the data, considering the period 2005 (or latest available) to 2019. The calibrated parameters for Peru and Uruguay are displayed in Table 2. The discount factor, β, is calibrated to target an average stock of external sovereign debt of 17% of GDP for the average Latin American country in our sample. For Peru and Uruguay the targets are 14% and 31%. The calibrated values are 0.960, 0.961 and 0.953, respectively. The benchmark rate of inflation, $\bar{\pi}$, is chosen to match the average inflation rate, which for the average country in the sample is 4.9%, for Peru is 2.8% and for Uruguay is 7.6%. The calibrated values are 1.043, 1.023 and 1.063, respectively. The decay rate of bonds, δ, is chosen to match the duration of bonds, which is 7 years for the sample average, 7.5 for Peru and 10.6 for Uruguay.[4] The calibrated values are 0.89, 0.90, and 0.95, respectively. The process for tradable endowment is estimated with annual data on tradable GDP (agricultural and manufacturing sectors) for the period 1990-2019 for the panel countries analyzed in section 2. The data of tradable GDP was detrended using a log-linear trend. The parameters that govern the bond issuance cost, $\psi_b, \bar{B}^*, \bar{B}, \bar{B}^\pi$, are calibrated in the following way. ψ_b is set to 0.065, chosen to be relatively small but at the same time accommodate the cases of Peru and Uruguay.[5] $\bar{B}^*, \bar{B}, \bar{B}^\pi$ are chosen to match average foreign currency, local currency and inflation-indexed shares in the average Latin American country, which are %67, 23% and 6%, respectively. The calibrated values are 0.22, 0.10 and 0.1, respectively.

For this model, the equilibrium policy functions cannot be computed and the model's steady state is endogenously determined as part of the model solution, so it is a priori unknown. Therefore, we cannot solve the model using a perturbation around the steady state. Following the solution method used by Bacchetta, Wincoop, and Young (2022), I use the Taylor projection method developed in Levintal (2018) to solve this problem. This involves approximating the solution locally at various nodes of the state space, and then combining these local solutions to form the global solution. (See brief description of the solution method in Appendix C)

[4]Total debt duration obtained from the ministry of finance of each country, and IMF's Article IV for Brazil.

[5]When adjusting β and δ to match the average debt-to-GDP ratio and duration of each country, we need to remember that $0 < \delta < \beta^{-1}$.

Table 3: Numerical Results

Moment	LAC Data	LAC Model	Peru Data	Peru Model	Uruguay Data	Uruguay Model
Average Level						
Debt to GDP	17%	17%	14%	14%	31%	31%
Share of debt in FC	67%	67%	66%	74%	82%	53%
Share of debt in LC	27%	27%	34%	26%	2%	28%
Share of indexed debt	6%	6%	0%	0%	16%	19%
Inflation	4.9%	4.9%	2.8%	2.8%	7.6%	7.6%
Correlation with GDP						
Inflation	-0.05	0.27	0.50	0.36	-0.51	-0.01
Nominal exchange rate	-0.52	-0.75	-0.65	-0.68	-0.83	-0.89
Real exchange rate	-0.52	-0.80	-0.67	-0.74	-0.90	-0.92

5.2 Numerical Results

Table 3 summarizes the data and model moments, calibrated for the average Latin American country (LAC), Peru and Uruguay. To compute the model's moments I simulate the exogenous stochastic process for tradable endowment for 100,000 periods, and discard the first 10,000 observations. Average level of total sovereign external debt to GDP and average inflation rate match the data counterpart, since there are targets of the calibration. Similarly, the currency composition of the external sovereign debt matches the data for the average Latin American country. However, when analyzing Peru and Uruguay I keep the parameters that govern the bond issuance cost, ψ_b, $\bar{B}^*$, $\bar{B}$ and $\bar{B}^\pi$, as calibrated for the average country in the sample. In the case of Peru, a smaller sovereign external debt-to-GDP, lower inflation rate and slightly higher bond duration relative to the average Latin American country lead to a smaller share of inflation-indexed debt and a larger share of foreign currency. On the contrary, in the case of Uruguay, a larger sovereign external debt-to-GDP, higher inflation rate and debt duration relative to the average Latin American country result in a larger share of inflation-indexed debt and a smaller share of foreign currency debt. The model predicts a 0 percent average share of inflation-indexed debt in the case of Peru, as observed in the data. For Uruguay, it predicts a 19 percent average share of inflation-indexed debt, close to the 16 percent observed in the data. Nevertheless, the model overestimates the observed share of foreign currency debt (and underestimates the share of local currency) in the case of Peru, and underestimates it in the case of Uruguay (and overestimates the share of local currency). The model correctly predicts a strong and negative correlation between exchange rate, nominal and real, and GDP.

- **Role of Average Debt, Duration and Inflation Costs**

To analyze these results, I consider the effect of each parameter individually. Figure 2 plots the outcomes for key parameters, considering the model calibrated to the average Latin American country as the baseline. An increase in the discount factor, β, all other things being equal, reduces the average external sovereign debt-to-GDP ratio. As analyzed in subsection 4.2, lower local currency and indexed debt levels, result in smaller hedging benefits, given the negative correlation between

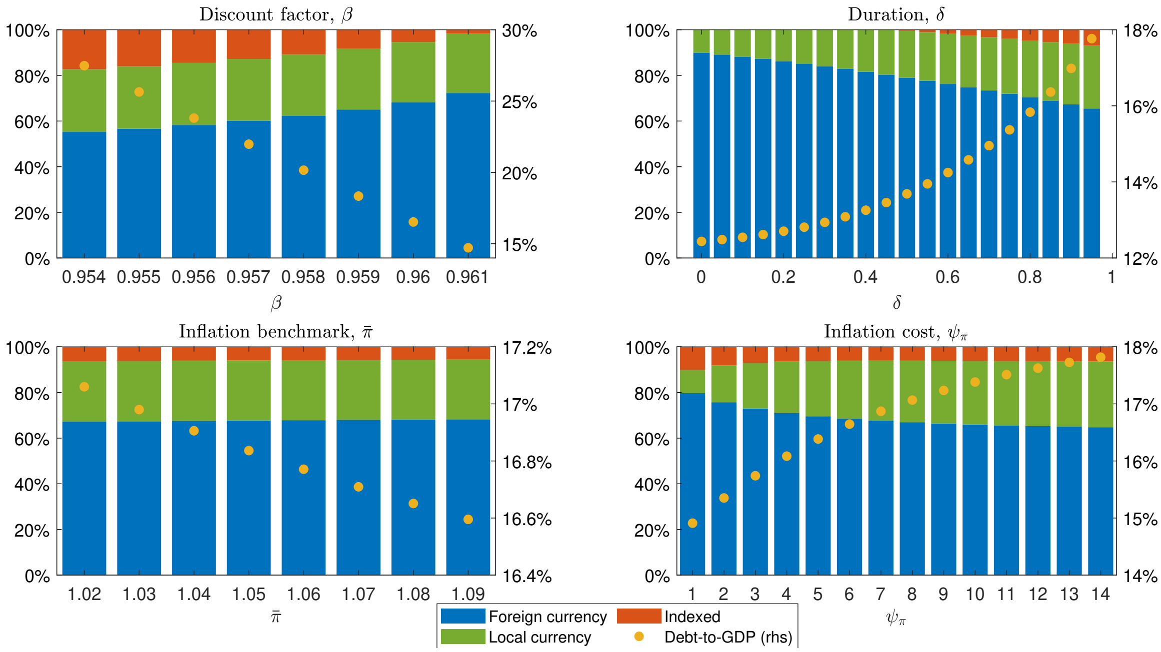

Figure 2: Currency composition (left-hand side axis) and average external debt-to-GDP (right-hand-side axis) for different parameter values. Except for the parameter subject to change, the calibration corresponds to the average Latin American country.

exchange rate, nominal and real, and tradable endowment. But at the same time, this reduces the government incentives to dilute the value of the debt ex-post through inflation and real exchange rate depreciation. This makes the extra return on local currency and inflation-indexed bonds required by foreign lenders smaller, and their prices higher. Additionally, as debt decreases so does the cost of issuing bonds introduced in subsection 5.1. As a result of balancing these benefits and costs, the share of inflation-indexed debt decreases when β increases, the share of foreign currency debt increases, while the share of local currency debt remains stable.

An increase in δ, all other things being equal, translates into an increase in debt duration.[6] As discussed in subsection 4.2, a larger duration increases the hedging benefits provided by local currency and inflation-indexed bonds, when there is a negative correlation between exchange rate, nominal and real, and tradable endowment. At the same time, a higher duration increases the benefits from diluting debt through surprise inflation. Therefore, the required return on bonds in local currency increases, and their prices decrease. As a result of balancing these benefits and costs, when δ rises, the share of local currency increases and the share of foreign currency debt decreases. For $\delta > 0.5$, i.e. duration larger than 2 years, as δ increases not only the share of local currency increases but also the share of inflation-indexed debt. Moreover, when duration increases, so does the average debt-to-GDP. Same logic applies as for a reduction in β.

Increases in inflation costs are governed by $\bar{\pi}$ and ψ_π, and affect the inflationary bias and the price of bonds, as explained in section 4.2. All other things being equal, the larger the inflation

[6] As mentioned in the model section (3), duration is given by $(1 - \beta\rho)^{-1}$.

Table 4: Numerical results changing one parameter at a time with LAC as the baseline

	LAC	Debt to GDP		Duration		Average inflation		Inflation cost	
Moment	Baseline	14%	31%	7.5y	10.6y	2.8%	7.6%	1.8	28.3
Average Level									
Debt to GDP	17%	14%	31%	17%	17%	17%	17%	17%	17%
Share of debt in FC	68%	73%	53%	67%	67%	67%	68%	72%	66%
Share of debt in LC	26%	26%	28%	27%	27%	27%	26%	15%	30%
Share of indexed debt	6%	1%	19%	6%	6%	6%	6%	12%	5%
Inflation	4.9%	4.9%	4.9%	4.9%	4.9%	2.8%	7.6%	4.9%	4.9%
Correlation with GDP									
Inflation	0.27	0.30	0.15	0.28	0.30	0.29	0.26	0.34	0.18
Nominal exchange rate	-0.75	-0.74	-0.77	-0.75	-0.76	-0.75	-0.79	-0.75	-0.76
Real exchange rate	-0.80	-0.79	-0.81	-0.80	-0.81	-0.80	-0.74	-0.80	-0.78

costs, the smaller the incentives for the government to generate ex-post inflation, and higher the price of bonds denominated in local currency (given the smaller extra return demanded by foreign lenders). Quantitatively, a rise in the inflation benchmark, $\bar{\pi}$, does not alter currency composition and it slightly reduces average debt-to-GDP ratio. An increase in ψ_π reduces the share of foreign currency and inflation-indexed debt, boosting the share of local currency bonds. This effect gets smaller as ψ_π increases. Additionally, the larger ψ_π, the larger the average debt-to-GDP ratio. Same logic applies as for a reduction in β.

Table 4 presents the numerical results changing one parameter at a time using the calibrated values for Peru and Uruguay. The first column refers to the results for the average Latin American country. Columns 2 to 3 show the effect of adjusting the calibration to match the average debt-to-GDP, duration and inflation rate of Peru and Uruguay. Even though each parameter has different effects on the currency composition, as noted in figure 1, in the case of Peru and Uruguay, the difference in the share of inflation-indexed debt is explained by the debt-to-GDP ratio. Column 4 introduces changes in the inflation cost parameter ψ_π, considering costs 2.5 times higher and smaller than the baseline. All other things being equal, the higher the inflation costs, the smaller the share of foreign currency and inflation-indexed debt. As inflation gets more costly, the incentives for the government to dilute the value of outstanding local currency debt get reduced. This lowers the extra return demanded by foreign lenders, and increases the price of local currency bonds. Therefore, the government can benefit from the hedging properties of local currency bonds, without paying a much higher return on them.

6 Conclusion

This paper examines empirically and theoretically the role of inflation-indexed sovereign external bonds as an additional policy tool for the government. I build a dataset on foreign holdings of government debt distinguishing between local currency, foreign currency and indexed bonds. In Latin American countries, the reduction in the external debt denominated in foreign currency experienced in the last two decades lead to an increase in the share of local currency debt. Inflation-indexed bonds represented a small fraction, with the exception of Uruguay.

I develop a model of optimal time-consistent monetary policy and choice of debt denominated in foreign currency, local currency and inflation-indexed units. The choice of debt currency denomination balances hedging benefits and costs derived from discipline effects, due to the ex-post incentive to dilute the real value of local currency and inflation-indexed bonds. I calibrate the model for the average Latin American country in the sample, Peru and Uruguay. According to the model, a larger debt-to-GDP ratio, longer debt duration and lower inflation costs encourage more borrowing in inflation-indexed bonds and can explain the larger share of inflation-indexed debt in Uruguay compared to other Latin American countries.

I leave for future research studying the optimal decision of debt duration and distortionary taxes. Over the last decades, duration has increased in Latin American countries, and exploring its brake-down by type of bonds can introduce interesting considerations for optimal policy. Distortionary taxes introduce an extra cost to altering intertemporal consumption decisions, and will affect hedging benefits and costs derived from dilution incentives. These two considerations are analyzed by Leeper, Leith, and D. Liu (2021) in a close economy framework, and it would be interesting to extend the analysis to an open economy.

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Appendix A Description of the Data

- **Brazil**. Foreign holdings of sovereign debt correspond to Federal Public Debt issued in domestic markets and held by nonresidents and debt issued in international markets, which is assumed to be held by nonresidents. The information was obtained from the annex of the monthly public debt report, elaborated by the National Treasury. For debt issued in international markets, data is available since January 2004. For debt issued in domestic markets, data is available since December 1999. However, its decomposition by bond-holder is available since January 2007. To obtain the information by bond-holder and currency denomination I created a dataset based on the information included in the monthly public debt reports, available since January 2011.

- **Colombia**. Foreign holdings of sovereign debt correspond to Central Government External Debt issued in domestic markets and held by nonresidents and debt issued in international markets, which are assumed to be held by nonresidents. For debt issued in domestic markets, the information was compiled based on Domestic Debt Profile Treasury Securities Reports, elaborated by the Ministry of Finance. I could gather this data since January 2010, but there is a discontinuity from January 2016 to December 2018. For debt issued in international markets, the information was obtained from the historical Central Government debt data, elaborated by the Ministry of Finance, and it is available since June 2001.

- **Mexico**. Foreign holdings of sovereign debt correspond to Public Sector Debt issued in domestic markets and held by nonresidents and debt issued in international markets, which is assumed to be held by nonresidents. The data about debt issued in international markets was obtained from the Federal Public Sector debt statistics elaborated by the Ministry of Finance since January 1990. The information about debt issued in domestic markets held by nonresidents was obtained from the Central Bank of Mexico, since January 1991.

- **Peru**. Foreign holdings of sovereign debt correspond to Non-Monetary Public Sector. Information about debt issued in international markets was obtained from the Public Debt Statistics elaborated by the Central Bank of Peru available since the first quarter of 1999. For debt issued in domestic markets, I constructed a dataset based on the Sovereign Bond Holding Reports elaborated by the General Directorate of the Public Treasury, Ministry of Economy and Finance. Information by currency was available since the first quarter of 2013.

- **Uruguay**. Foreign holdings of sovereign debt correspond to Non-Monetary Public Sector. Debt held by nonresidents was obtained from the Central Bank of Uruguay, available since the last quarter of 1999. To distinguish between local currency and inflation-indexed debt I assume that external debt (60% of total debt) is split as total debt. Before 2012 there was no debt denominated in local currency, not issued in domestic nor international markets.

In all possible cases I verify that the data is similar to Arslanalp and Tsuda (2014).

Appendix B Optimal Policy Derivations

B.1 Optimal Policy under Commitment and Discretion

B.1.1 Commitment

When the government can commit to future policies, the Lagrangian for the policy problem is given by

$$
\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{ & u\left(C\left(c_{T,t}, y_N\right)\right) - l(\pi_t) \\
& + \lambda_t \left[y_{T,t} - c_{T,t} - n\left(\pi_t, \frac{c_{T,t}}{y_N}\right)(1+\delta Q_t) b_{t-1} - n\left(1, \frac{c_{T,t}}{y_N}\right)(1+\delta Q_t^\pi) b_{t-1}^\pi - \frac{R}{R-\delta} b_{t-1}^* \right. \\
& \left. + n\left(1, \frac{c_{T,t}}{y_N}\right) Q_t b_t + n\left(1, \frac{c_{T,t}}{y_N}\right) Q_t^\pi b_t^\pi + \frac{1}{R-\delta} b_t^* \right] \\
& + \mu_t \left\{ Q_t - \frac{1}{R} \frac{1}{n\left(1, \frac{c_{T,t}}{y_N}\right)} E_t \left[n\left(\pi_{t+1}, \frac{c_{T,t+1}}{y_N}\right)(1+\delta Q_{t+1}) \right] \right\} \\
& + \mu_t^\pi \left\{ Q_t^\pi - \frac{1}{R} \frac{1}{n\left(1, \frac{c_{T,t}}{y_N}\right)} E_t \left[n\left(1, \frac{c_{T,t+1}}{y_N}\right)(1+\delta Q_{t+1}^\pi) \right] \right\} \}
\end{aligned}
\tag{31}
$$

By committing to an entire path of policy instruments, the government is able to influence expectations in order to improve the policy trade-offs they face.

The resultant set of FOCs are given by

$$
\begin{aligned}
[c_{T,t}]: \quad & u'(c_t)C_{c_T,t} \\
& -\lambda_t\left[1+n_c\left(\pi_t,\frac{c_{T,t}}{y_N}\right)(1+\delta Q_t)b_{t-1}-n_c\left(1,\frac{c_{T,t}}{y_N}\right)(1+\delta Q_t^\pi)b_{t-1}^\pi-n_c\left(1,\frac{c_{T,t}}{y_N}\right)(Q_tb_t+Q_t^\pi b_t^\pi)\right] \\
& +\mu_t\frac{1}{R}\frac{n_c\left(1,\frac{c_{T,t}}{y_N}\right)}{n\left(1,\frac{c_{T,t}}{y_N}\right)^2}E_t\left[n\left(\pi_{t+1},\frac{c_{T,t+1}}{y_N}\right)(1+\delta Q_{t+1})\right] \\
& +\mu_t^\pi\frac{1}{R}\frac{n_c\left(1,\frac{c_{T,t}}{y_N}\right)}{n\left(1,\frac{c_{T,t}}{y_N}\right)^2}E_t\left[n\left(1,\frac{c_{T,t+1}}{y_N}\right)(1+\delta Q_{t+1}^\pi)\right] \\
& -\beta^{-1}\mu_{t-1}\frac{1}{R}\frac{1}{n\left(1,\frac{c_{T,t-1}}{y_N}\right)}n_c\left(\pi_t,\frac{c_{T,t}}{y_N}\right)(1+\delta Q_t) \\
& -\beta^{-1}\mu_{t-1}^\pi\frac{1}{R}\frac{1}{n\left(1,\frac{c_{T,t-1}}{y_N}\right)}n_c\left(1,\frac{c_{T,t}}{y_N}\right)(1+\delta Q_t^\pi)=0 \\
[\pi_t]: \quad & -l'(\pi_t)-\lambda_t n_\pi\left(\pi_t,\frac{c_{T,t}}{y_N}\right)(1+\delta Q_t)b_{t-1}-\frac{1}{\beta}\mu_{t-1}\frac{1}{R}\frac{1}{n\left(1,\frac{c_{T,t-1}}{y_N}\right)}n_\pi\left(\pi_t,\frac{c_{T,t}}{y_N}\right)(1+\delta Q_t)=0 \\
[b_t]: \quad & \lambda_t n\left(1,\frac{c_{T,t}}{y_N}\right)Q_t-\beta E_t\left[\lambda_{t+1}n\left(\pi_{t+1},\frac{c_{T,t+1}}{y_N}\right)(1+\delta Q_{t+1})\right]=0 \\
[b_t^\pi]: \quad & \lambda_t n\left(1,\frac{c_{T,t}}{y_N}\right)Q_t^\pi-\beta E_t\left[\lambda_{t+1}n\left(1,\frac{c_{T,t+1}}{y_N}\right)(1+\delta Q_{t+1}^\pi)\right]=0 \\
[b_t^*]: \quad & \lambda_t\frac{1}{R-\delta}-\beta E_t\left[\lambda_{t+1}\right]\frac{R}{R-\delta}=0\Rightarrow\lambda_t-\beta E_t\left[\lambda_{t+1}\right]R=0 \\
[Q_t]: \quad & -\lambda_t\left[n\left(\pi_t,\frac{c_{T,t}}{y_N}\right)\delta b_{t-1}+n\left(1,\frac{c_{T,t}}{y_N}\right)b_t\right]+\mu_t-\beta^{-1}\mu_{t-1}\frac{1}{R}\frac{1}{n\left(1,\frac{c_{T,t-1}}{y_N}\right)}n\left(\pi_t,\frac{c_{T,t}}{y_N}\right)\delta=0 \\
[Q_t^\pi]: \quad & -\lambda_t\left[n\left(1,\frac{c_{T,t}}{y_N}\right)\delta b_{t-1}^\pi+n\left(1,\frac{c_{T,t}}{y_N}\right)b_t^\pi\right]+\mu_t^\pi-\beta^{-1}\mu_{t-1}^\pi\frac{1}{R}\frac{1}{n\left(1,\frac{c_{T,t-1}}{y_N}\right)}n\left(1,\frac{c_{T,t}}{y_N}\right)\delta=0
\end{aligned}
\tag{32}
$$

B.1.2 Discretion

The Lagrangian for the policy problem under discretion is given by

$$
\begin{aligned}
\mathcal{L}= & \, u(C(c_{T,t},y_N))-l(\pi_t)+\beta E_t[V(s_{t+1})] \\
& +\lambda_t\left\{y_{T,t}-c_{T,t}-b_{t-1}^*-n\left(\pi_t,\frac{c_{T,t}}{y_N}\right)b_{t-1}-n\left(1,\frac{c_{T,t}}{y_N}\right)b_{t-1}^\pi+\frac{1}{R-\delta}\left(b_t^*-\delta b_{t-1}^*\right)\right. \\
& \left.+\frac{1}{R}E_t\left[\mathcal{X}(s_{t+1})\left(b_t-\delta\frac{b_{t-1}}{\pi_t}\right)+\mathcal{Z}(s_{t+1})\left(b_t^\pi-\delta b_{t-1}^\pi\right)\right]\right\}
\end{aligned}
\tag{33}
$$

where the model equilibrium also involves the bond prices definitions given by (15) and (16).

The FOCs for the policy problem are

$$
\begin{aligned}
[c_{T,t}]: &\quad u'(c_t)C_{c_T,t} - \lambda_t \left[1 + n_c\left(\pi_t, \frac{c_{T,t}}{y_N}\right) b_{t-1} + n_c\left(1, \frac{c_{T,t}}{y_N}\right) b^{\pi}_{t-1}\right] = 0 \\
[\pi_t]: &\quad -l'(\pi_t) - \lambda_t \left\{ n_\pi\left(\pi_t, \frac{c_{T,t}}{y_N}\right) - \delta\frac{1}{R}E_t[\mathcal{X}(s_{t+1})]\frac{1}{\pi_t^2}\right\} b_{t-1} = 0 \\
[b^*_t]: &\quad \beta E_t\left[\lambda_{t+1}\right]\left(1 + \frac{\delta}{R-\delta}\right) \\
&\quad - \lambda_t \left\{\frac{1}{R-\delta} + \frac{1}{R}E_t[\mathcal{X}_{b^*}(s_{t+1})]\left(b_t - \delta\frac{b_{t-1}}{\pi}\right) + \frac{1}{R}E_t[\mathcal{Z}_{b^*}(s_{t+1})]\left(b^{\pi}_t - \delta b^{\pi}_{t-1}\right)\right\} = 0 \\
[b_t]: &\quad \beta E_t\left[\lambda_{t+1}\mathcal{X}(s_{t+1})\right] \\
&\quad - \lambda_t \frac{1}{R}\left\{E[\mathcal{X}_b(s_{t+1})]\left(b_t - \delta\frac{b_{t-1}}{\pi_t}\right) + E_t[\mathcal{Z}_b(s_{t+1})]\left(b^{\pi}_t - \delta b^{\pi}_{t-1}\right) + E_t[\mathcal{X}(s_{t+1})]\right\} = 0 \\
[b^{\pi}_t]: &\quad \beta E_t\left[\lambda_{t+1}\mathcal{Z}(s_{t+1})\right] \\
&\quad - \lambda_t \frac{1}{R}\left\{E_t[\mathcal{X}_{b^{\pi}}(s_{t+1})]\left(b_t - \delta\frac{b_{t-1}}{\pi_t}\right) + E_t[\mathcal{Z}_{b^{\pi}}(s_{t+1})]\left(b^{\pi}_t - \delta b^{\pi}_{t-1}\right) + E_t[\mathcal{Z}(s_{t+1})]\right\} = 0
\end{aligned} \tag{34}
$$

Note that from the FOC with respect to consumption we obtain:

$$\lambda_t = \frac{u'(c_t)C_{c_T,t}}{1 + n_c(\pi_t, c_{T,t})b_{t-1} + n_c(1, c_{T,t})b^{\pi}_{t-1}} \tag{35}$$

B.2 Generalized Euler Equations

The trade-offs associated to the government choices arise from the analysis of the fist order conditions. Define $\mathcal{C}(b^*_t, b_t, b^{\pi}_t), \mathcal{P}(b^*_t, b_t, b^{\pi}_t), \mathcal{B}^*(b^*_t, b_t, b^{\pi}_t), \mathcal{B}(b^*_t, b_t, b^{\pi}_t), \mathcal{B}^{\pi}(b^*_t, b_t, b^{\pi}_t)$ the expected tradable consumption, inflation, and debt policies in foreign currency, local currency, and inflation-indexed units, respectively. In equilibrium, these expectations are consistent with optimal policies. Without loss of generality we set $y_N = 1$, and assume y_T is constant. The recursive government problem can be expressed as

$$V(b^*_{t-1}, b_{t-1}, b^{\pi}_{t-1}) = \max_{c_{T,t},\pi_t,b^*_t,b_t,b^{\pi}_t} u\left(C\left(c_{T,t}, 1\right)\right) - l(\pi_t) + \beta E_t\left[V(b^*_t, b_t, b^{\pi}_t)\right] \tag{36}$$

subject to the household's budget constraint (3), the government's budget constraint (5) after using bond prices (6-8) and the inverse of the nominal exchange rate function (10):

$$
\begin{aligned}
&-y_{T,t} - c_{T,t} - b^*_{t-1} - n\left(\pi_t, c_{T,t}\right) b_{t-1} - n\left(1, c_{T,t}\right) b^{\pi}_{t-1} + \frac{1}{R}b^*_t \\
&\quad + \frac{1}{R}E_t\left[n\left(\mathcal{P}(b_t, b^{\pi}_t, b^*_t), \mathcal{C}(b_t, b^{\pi}_t, b^*_t)\right) b_t + n\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^*_t)\right) b^{\pi}_t\right] = 0
\end{aligned} \tag{37}
$$

The FOCs of this problem are given by

$$
\begin{aligned}
&[c_t] : u'(c_t)C_{c_T,t} &&= \lambda_t + \lambda_t n_c(\pi_t, c_{T,t})b_{t-1} + \lambda_t n_c(1, c_{T,t})b^{\pi}_{t-1} && (38)\\
&[\pi_t] : -l'(\pi_t) &&= \lambda_t n_\pi(\pi_t, c_{T,t})b_{t-1} && (39)\\
&[b_t] : \beta V_b(b_t, b^\pi_t, b^*_t) &&= -\lambda_t \frac{1}{R}\left\{\frac{\partial[n\left(\mathcal{P}(b_t, b^\pi_t, b^*_t), \mathcal{C}(b_t, b^\pi_t, b^*_t)\right) b_t]}{\partial b_t} + \frac{\partial n\left(1, \mathcal{C}(b_t, b^\pi_t, b^*_t)\right)}{\partial b_t} b^\pi_t\right\} && (40)\\
&[b^\pi_t] : \beta V_{b^\pi}(b_t, b^\pi_t, b^*_t) &&= -\lambda_t \frac{1}{R}\left\{\frac{\partial n\left(\mathcal{P}(b_t, b^\pi_t, b^*_t), \mathcal{C}(b_t, b^\pi_t, b^*_t)\right)}{\partial b^\pi_t} b_t + \frac{\partial[n\left(1, \mathcal{C}(b_t, b^\pi_t, b^*_t)\right) b^\pi_t]}{\partial b^\pi_t}\right\} && (41)\\
&[b^*_t] : \beta V_{b^*}(b_t, b^\pi_t, b^*_t) &&= -\lambda_t \frac{1}{R}\left[1 + \frac{\partial n\left(\mathcal{P}(b_t, b^\pi_t, b^*_t), \mathcal{C}(b_t, b^\pi_t, b^*_t)\right)}{\partial b^*_t} b_t + \frac{\partial n\left(1, \mathcal{C}(b_t, b^\pi_t, b^*_t)\right)}{\partial b^*_t} b^\pi_t\right] && (42)
\end{aligned}
$$

and the envelope conditions are

$$
\begin{aligned}
&[b_{t-1}] : \beta V_b(b_{t-1}, b^\pi_{t-1}, b^*_{t-1}) &&= -\lambda_t n(\pi_t, c_{T,t}) && (43)\\
&[b^\pi_{t-1}] : \beta V_{b^\pi}(b_{t-1}, b^\pi_{t-1}, b^*_{t-1}) &&= -\lambda_t n(1, c_{T,t}) && (44)\\
&[b^*_{t-1}] : \beta V_{b^*}(b_t, b^\pi_{t-1}, b^*_{t-1}) &&= -\lambda_t && (45)
\end{aligned}
$$

Then

$$
\lambda_t = \frac{u'(c_t)C_{c_T,t}}{1 + n_c(\pi_t, c_{T,t})b_{t-1} + n_c(1, c_{T,t})b^\pi_{t-1}} \tag{46}
$$

Combining (46) with the envelope conditions we obtain three Euler equations:

$$
\begin{aligned}
\lambda_t \left\{\frac{\partial[n\left(\mathcal{P}(b_t, b^\pi_t, b^*_t), \mathcal{C}(b_t, b^\pi_t, b^*_t)\right) b_t]}{\partial b_t} + \frac{\partial n\left(1, \mathcal{C}(b_t, b^\pi_t, b^*_t)\right)}{\partial b_t} b^\pi_t\right\} &= \beta R \lambda_{t+1} n(\pi_{t+1}, c_{T,t+1}) && (47)\\
\lambda_t \left\{\frac{\partial n\left(\mathcal{P}(b_t, b^\pi_t, b^*_t), \mathcal{C}(b_t, b^\pi_t, b^*_t)\right)}{\partial b^\pi_t} b_t + \frac{\partial[n\left(1, \mathcal{C}(b_t, b^\pi_t, b^*_t)\right) b^\pi_t]}{\partial b^\pi_t}\right\} &= \beta R \lambda_{t+1} n(1, c_{T,t+1}) && (48)\\
\lambda_t \left[1 + \frac{\partial n\left(\mathcal{P}(b_t, b^\pi_t, b^*_t), \mathcal{C}(b_t, b^\pi_t, b^*_t)\right)}{\partial b^*_t} b_t + \frac{\partial n\left(1, \mathcal{C}(b_t, b^\pi_t, b^*_t)\right)}{\partial b^*_t} b^\pi_t\right] &= \beta R \lambda_{t+1} && (49)
\end{aligned}
$$

The price sensitivity of debt can be calculated. The resource constraint of this economy at $t+1$ is

$$
\begin{aligned}
&\mathcal{C}(b_t, b^\pi_t, b^*_t) - y_{T,t+1} - n\left(\mathcal{P}(b_t, b^\pi_t, b^*_t), \mathcal{C}(b_t, b^\pi_t, b^*_t)\right) b_t - n\left(1, \mathcal{C}(b_t, b^\pi_t, b^*_t)\right) b^\pi_t - b^*_t + \frac{1}{R}\mathcal{B}^*(b_t, b^\pi_t, b^*_t)\\
&+ \frac{1}{R} E_t\left[n\left(\mathcal{P}(\mathcal{B}(b_t, b^\pi_t, b^*_t), \mathcal{B}^\pi(b_t, b^\pi_t, b^*_t), \mathcal{B}^*(b_t, b^\pi_t, b^*_t)), \mathcal{C}(\mathcal{B}(b_t, b^\pi_t, b^*_t), \mathcal{B}^\pi(b_t, b^\pi_t, b^*_t), \mathcal{B}^*(b_t, b^\pi_t, b^*_t))\right) \mathcal{B}(b_t, b^\pi_t, b^*_t)\right]\\
&+ \frac{1}{R} E_t\left[n\left(1, \mathcal{C}(\mathcal{B}(b_t, b^\pi_t, b^*_t), \mathcal{B}^\pi(b_t, b^\pi_t, b^*_t), \mathcal{B}^*(b_t, b^\pi_t, b^*_t))\right) \mathcal{B}^\pi(b_t, b^\pi_t, b^*_t)\right] = 0
\end{aligned} \tag{50}
$$

To simplify notation define

$$
\begin{aligned}
&n(\pi_{t+2}, c_{T,t+2})b_{t+1} + n(1, c_{T,t+2})b^{\pi}_{t+1} + b^{*}_{t+1} \equiv \\
&n\left(\mathcal{P}(\mathcal{B}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{B}^{\pi}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{B}^{*}(b_t, b^{\pi}_t, b^{*}_t)), \mathcal{C}(\mathcal{B}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{B}^{\pi}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{B}^{*}(b_t, b^{\pi}_t, b^{*}_t))\right) \mathcal{B}(b_t, b^{\pi}_t, b^{*}_t) \\
&+ n\left(1, \mathcal{C}(\mathcal{B}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{B}^{\pi}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{B}^{*}(b_t, b^{\pi}_t, b^{*}_t))\right) \mathcal{B}^{\pi}(b_t, b^{\pi}_t, b^{*}_t) + \mathcal{B}^{*}(b_t, b^{\pi}_t, b^{*}_t)
\end{aligned}
\tag{51}
$$

Differentiating the resource constraint at $t+1$ with respect to b_t, b^{π}_t and b^{*}_t:

$$
\begin{aligned}
\frac{\partial \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)}{\partial b_t} = &- \frac{\partial [n\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) b_t]}{\partial b_t} - \frac{\partial n\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right)}{\partial b_t} b^{\pi}_t \\
&+ \frac{1}{R} \frac{\partial [n(\pi_{t+2}, c_{T,t+2})b_{t+1} + n(1, c_{T,t+2})b^{\pi}_{t+1} + b^{*}_{t+1}]}{\partial b_t}
\end{aligned}
\tag{52}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)}{\partial b^{\pi}_t} = &- \frac{\partial n\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right)}{\partial b^{\pi}_t} b_t - \frac{\partial [n\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) b^{\pi}_t]}{\partial b^{\pi}_t} \\
&+ \frac{1}{R} \frac{\partial [n(\pi_{t+2}, c_{T,t+2})b_{t+1} + n(1, c_{T,t+2})b^{\pi}_{t+1} + b^{*}_{t+1}]}{\partial b^{\pi}_t}
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)}{\partial b^{*}_t} = &- \frac{\partial n\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right)}{\partial b^{*}_t} b_t - \frac{\partial n\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right)}{\partial b^{*}_t} b^{\pi}_t - 1 \\
&+ \frac{1}{R} \frac{\partial [n(\pi_{t+2}, c_{T,t+2})b_{t+1} + n(1, c_{T,t+2})b^{\pi}_{t+1} + b^{*}_{t+1}]}{\partial b^{*}_t}
\end{aligned}
\tag{54}
$$

Applying the chain rule to the first two terms of the previous equations

$$
\begin{aligned}
\frac{\partial [n\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) b_t]}{\partial b_t} = &\; n\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) \\
&+ b_t n_{\pi}\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) \mathcal{P}_b(b_t, b^{\pi}_t, b^{*}_t) \\
&+ b_t n_c\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) \mathcal{C}_b(b_t, b^{\pi}_t, b^{*}_t)
\end{aligned}
\tag{55}
$$

$$
\frac{\partial n\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right)}{\partial b_t} = n_c\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) \mathcal{C}_b(b_t, b^{\pi}_t, b^{*}_t)
\tag{56}
$$

$$
\begin{aligned}
\frac{\partial n\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right)}{\partial b^{\pi}_t} = &\; n_{\pi}\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) \mathcal{P}_{b^{\pi}}(b_t, b^{\pi}_t, b^{*}_t) \\
&+ n_c\left(\mathcal{P}(b_t, b^{\pi}_t, b^{*}_t), \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) \mathcal{C}_{b^{\pi}}(b_t, b^{\pi}_t, b^{*}_t)
\end{aligned}
\tag{57}
$$

$$
\frac{\partial [n\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) b^{\pi}_t]}{\partial b^{\pi}_t} = n\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) + b^{\pi}_t n_c\left(1, \mathcal{C}(b_t, b^{\pi}_t, b^{*}_t)\right) \mathcal{C}_{b^{\pi}}(b_t, b^{\pi}_t, b^{*}_t)
\tag{58}
$$

$$\begin{aligned}\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*} &= n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^*}(b_t,b_t^\pi,b_t^*)\\ &+ n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{C}_{b^*}(b_t,b_t^\pi,b_t^*)\end{aligned} \tag{59}$$

$$\frac{\partial n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*} = n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{C}_{b^*}(b_t,b_t^\pi,b_t^*) \tag{60}$$

To obtain the Euler equation for debt in local currency, we first combine equations (52), (55) and (56)

$$\begin{aligned}&\frac{\partial[n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)b_t]}{\partial b_t} + \frac{\partial n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t}b_t^\pi =\\ &n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_b(b_t,b_t^\pi,b_t^*)\\ &+ b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{C}_b(b_t,b_t^\pi,b_t^*) + b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{C}_b(b_t,b_t^\pi,b_t^*) =\\ &n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_b(b_t,b_t^\pi,b_t^*)\\ &+ \left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\mathcal{C}_b(b_t,b_t^\pi,b_t^*) =\\ &n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_b(b_t,b_t^\pi,b_t^*)\\ &- \left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{\partial[n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)b_t]}{\partial b_t}\\ &- \left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{\partial m\left(\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t}b_t^\pi\\ &+ \left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{1}{R}\frac{\partial[n(\pi_{t+2},c_{T,t+2})b_{t+1} + n(1,c_{T,t+2})b_{t+1}^\pi + b_{t+1}^*]}{\partial b_t}\end{aligned} \tag{61}$$

Rearranging terms we get

$$\begin{aligned}&\frac{\partial[n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)b_t]}{\partial b_t} + \frac{\partial n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t}b_t^\pi =\\ &\frac{1}{1 + b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}\left\{n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right.\\ &+ b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_b(b_t,b_t^\pi,b_t^*)\\ &\left.+ \left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right) + b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{1}{R}\frac{\partial[n(\pi_{t+2},c_{T,t+2})b_{t+1} + n(1,c_{T,t+2})b_{t+1}^\pi + b_{t+1}^*]}{\partial b_t}\right\}\end{aligned} \tag{62}$$

Finally, we substitute this equation into (47), to obtain (using (46)) the modified Euler equation for

debt in local currency

$$
\begin{aligned}
\frac{u'(c_t)C_{c_T,t}}{1+n_c(\pi_t,c_{T,t})b_{t-1}+n_c(1,c_{T,t})b^{\pi}_{t-1}} &= \beta R \frac{u'(c_{t+1})C_{c_T,t+1}}{1+n_c(\pi_{t+1},c_{T,t+1})b_t+n_c(1,c_{T,t+1})b^{\pi}_t} \\
&\quad n(\pi_{t+1},c_{T,t+1})\left\{\frac{\partial[n\left(\mathcal{P}(b_t,b^{\pi}_t,b^*_t),\mathcal{C}(b_t,b^{\pi}_t,b^*_t)\right)b_t]}{\partial b_t}+\frac{\partial n\left(1,\mathcal{C}(b_t,b^{\pi}_t,b^*_t)\right)}{\partial b_t}b^{\pi}_t\right\}^{-1} \\
&= \beta R u'(c_{t+1})C_{c_T,t+1} \\
&\quad \frac{1}{1+\frac{b_t n_{\pi}(t+1)\pi_b(t+1)+[b_t n_c(t+1)+b^{\pi}_t m_c(t+1)]\frac{1}{R}\frac{\partial\left[n(t+2)b_{t+1}+m(t+2)b^{\pi}_{t+1}+b^*_{t+1}\right]}{\partial b_t}}{n(t+1)}}
\end{aligned}
\tag{63}
$$

where $m(t)=n(1,c_{T,t})$.

So

$$
\begin{aligned}
u'_t C_{c_T,t} = \beta R u'_{t+1} C_{c_T,t+1} & \underbrace{\left(1+n_{c_T,t}b_{t-1}+m_{c,t}b^{\pi}_{t-1}\right)}_{\text{Dilution through RER}} \\
& \underbrace{\frac{1}{1+\frac{b_t n_{\pi,t+1}\pi_{b,t+1}+(b_t n_{c_T,t+1}+b^{\pi}_t m_{c,t+1})\frac{1}{R}\frac{\partial\left(n_{t+2}b_{t+1}+m_{t+2}b^{\pi}_{t+1}+b^*_{t+1}\right)}{\partial b_t}}{n_{t+1}}}}_{\text{Discipline effect}}
\end{aligned}
\tag{64}
$$

where, to simplify notation, $f(x_t,y_t)=f_t$ and $\partial f(x_t,y_t)/\partial x_t=f_{x,t}$.

To obtain the Euler equation for inflation-indexed debt, we first combine equations (53), (57) and (58)

$$
\begin{aligned}
&\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^\pi}b_t+\frac{\partial[n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)b_t^\pi]}{\partial b_t^\pi}=\\
&b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^\pi}(b_t,b_t^\pi,b_t^*)\\
&+b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{C}_{b^\pi}(b_t,b_t^\pi,b_t^*)\\
&+n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{C}_{b^\pi}(b_t,b_t^\pi,b_t^*)=\\
&b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^\pi}(b_t,b_t^\pi,b_t^*)+n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\\
&+\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\mathcal{C}_{b^\pi}(b_t,b_t^\pi,b_t^*)=\\
&b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^\pi}(b_t,b_t^\pi,b_t^*)+n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\\
&-\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^\pi}b_t\\
&-\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{\partial[n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)b_t^\pi]}{\partial b_t^\pi}\\
&+\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{1}{R}\frac{\partial[n(\pi_{t+2},c_{T,t+2})b_{t+1}+n(1,c_{T,t+2})b_{t+1}^\pi+b_{t+1}^*]}{\partial b_t^\pi}=\\
&b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^\pi}(b_t,b_t^\pi,b_t^*)+n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\\
&-\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\left\{\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^\pi}b_t+\frac{\partial[n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)b_t^\pi]}{\partial b_t^\pi}\right\}\\
&+\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{1}{R}\frac{\partial[n(\pi_{t+2},c_{T,t+2})b_{t+1}+n(1,c_{T,t+2})b_{t+1}^\pi+b_{t+1}^*]}{\partial b_t^\pi}
\end{aligned}
\tag{65}
$$

Rearranging terms we get

$$
\begin{aligned}
&\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^\pi}b_t+\frac{\partial[n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)b_t^\pi]}{\partial b_t^\pi}=\frac{1}{1+b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}\\
&\{b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^\pi}(b_t,b_t^\pi,b_t^*)+n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+\\
&+\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{1}{R}\frac{\partial[n(\pi_{t+2},c_{T,t+2})b_{t+1}+n(1,c_{T,t+2})b_{t+1}^\pi+b_{t+1}^*]}{\partial b_t^\pi}\right\}
\end{aligned}
\tag{66}
$$

Finally, we substitute this equation into (48), to obtain (using (46)) the modified Euler equation for

inflation-indexed debt

$$
\begin{aligned}
\frac{u'(c_t)C_{c_T,t}}{1+n_c(\pi_t,c_{T,t})b_{t-1}+n_c(1,c_{T,t})b_{t-1}^{\pi}} = \beta R \frac{u'(c_{t+1})C_{c_T,t+1}}{1+n_c(\pi_{t+1},c_{T,t+1})b_t+n_c(1,c_{T,t+1})b_t^{\pi}} \\
n(1,c_{T,t+1})\left\{\frac{\partial n\left(\mathcal{P}(b_t,b_t^{\pi},b_t^*),\mathcal{C}(b_t,b_t^{\pi},b_t^*)\right)}{\partial b_t^{\pi}}b_t+\frac{\partial[n\left(1,\mathcal{C}(b_t,b_t^{\pi},b_t^*)\right)b_t^{\pi}]}{\partial b_t^{\pi}}\right\}^{-1} \\
= \beta R u'(c_{t+1})C_{c_T,t+1} \\
\frac{1}{1+\frac{b_t n_{\pi}(t+1)\pi_{b^{\pi}}(t+1)+[b_t n_c(t+1)+b_t^{\pi} m_c(t+1)]\frac{1}{R}\frac{\partial\left[n(t+2)b_{t+1}+m(t+2)b_{t+1}^{\pi}+b_{t+1}^*\right]}{\partial b_t^{\pi}}}{m(t+1)}}
\end{aligned}
\tag{67}
$$

So

$$
\begin{aligned}
u_t'C_{c_T,t} = \beta R u_{t+1}' C_{c_T,t+1}\underbrace{\left(1+n_{c_T,t}b_{t-1}+m_{c,t}b_{t-1}^{\pi}\right)}_{\text{Dilution through RER}} \\
\underbrace{\frac{1}{1+\frac{b_t n_{\pi,t+1}\pi_{b^{\pi},t+1}+(b_t n_{c_T,t+1}+b_t^{\pi} m_{c,t+1})\frac{1}{R}\frac{\partial\left(n_{t+2}b_{t+1}+m_{t+2}b_{t+1}^{\pi}+b_{t+1}^*\right)}{\partial b_t^{\pi}}}{m_{t+1}}}}_{\text{Discipline effect}}
\end{aligned}
\tag{68}
$$

Finally, to obtain the Euler equation for debt in foreign currency, we first combine equations

(54), (59) and (60)

$$
\begin{aligned}
&1+\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*}b_t+\frac{\partial n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*}b_t^\pi=1+b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^*}(b_t,b_t^\pi,b_t^*)\\
&+b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{C}_{b^*}(b_t,b_t^\pi,b_t^*)+\\
&b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{C}_{b^*}(b_t,b_t^\pi,b_t^*)\\
&=1+b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^*}(b_t,b_t^\pi,b_t^*)\\
&+\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\mathcal{C}_{b^*}(b_t,b_t^\pi,b_t^*)\\
&=1+b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^*}(b_t,b_t^\pi,b_t^*)\\
&-\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*}b_t\\
&-\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{\partial n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*}b^\pi\\
&-\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\\
&+\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{1}{R}\frac{\partial[n(\pi_{t+2},c_{T,t+2})b_{t+1}+n(1,c_{T,t+2})b_{t+1}^\pi+b_{t+1}^*]}{\partial b_t^*}\\
&=1+b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^*}(b_t,b_t^\pi,b_t^*)\\
&-\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\left[1+\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*}b_t+\frac{\partial n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*}b_t^\pi\right]\\
&+\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{1}{R}\frac{\partial[n(\pi_{t+2},c_{T,t+2})b_{t+1}+n(1,c_{T,t+2})b_{t+1}^\pi+b_{t+1}^*]}{\partial b_t^*}
\end{aligned}
\tag{69}
$$

Rearranging terms we get

$$
\begin{aligned}
&1+\frac{\partial n\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*}b_t+\frac{\partial n\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}{\partial b_t^*}b_t^\pi=\frac{1}{1+b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)}\\
&\Big\{1+b_t n_\pi\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\mathcal{P}_{b^*}(b_t,b_t^\pi,b_t^*)\\
&+\left[b_t n_c\left(\mathcal{P}(b_t,b_t^\pi,b_t^*),\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)+b_t^\pi n_c\left(1,\mathcal{C}(b_t,b_t^\pi,b_t^*)\right)\right]\frac{1}{R}\frac{\partial[n(\pi_{t+2},c_{T,t+2})b_{t+1}+n(1,c_{T,t+2})b_{t+1}^\pi+b_{t+1}^*]}{\partial b_t^*}\Big\}
\end{aligned}
\tag{70}
$$

Finally, we substitute this equation into (49), to obtain (using (46)) the modified Euler equation for

debt in foreign currency

$$\frac{u'(c_t)C_{c_T,t}}{1+n_c(\pi_t,c_{T,t})b_{t-1}+n_c(1,c_{T,t})b^\pi_{t-1}} = \beta R\frac{u'(c_{t+1})C_{c_T,t+1}}{1+n_c(\pi_{t+1},c_{T,t+1})b_t+n_c(1,c_{T,t+1})b^\pi_t}$$
$$\left\{1+\frac{\partial n\left(\mathcal{P}(b_t,b^\pi_t,b^*_t),\mathcal{C}(b_t,b^\pi_t,b^*_t)\right)}{\partial b^*_t}b_t+\frac{\partial n\left(1,\mathcal{C}(b_t,b^\pi_t,b^*_t)\right)}{\partial b^*_t}b^\pi_t\right\}^{-1}$$
$$= \beta R u'(c_{t+1})C_{c_T,t+1}$$
$$\frac{1}{1+b_t n_\pi(t+1)\pi_{b^*}(t+1)+[b_t n_c(t+1)+b^\pi_t m_c(t+1)]\frac{1}{R}\frac{\partial\left[n(t+2)b_{t+1}+m(t+2)b^\pi_{t+1}+b^*_{t+1}\right]}{\partial b^*_t}} \tag{71}$$

So

$$u'_t C_{c_T,t} = \beta R u'_{t+1}C_{c_T,t+1}\underbrace{\left(1+n_{c_T,t}b_{t-1}+m_{c,t}b^\pi_{t-1}\right)}_{\text{Dilution through RER}}$$
$$\underbrace{\frac{1}{1+b_t n_{\pi,t+1}\pi_{b^*,t+1}+(b_t n_{c_T,t+1}+b^\pi_t m_{c,t+1})\frac{1}{R}\frac{\partial\left(n_{t+2}b_{t+1}+m_{t+2}b^\pi_{t+1}+b^*_{t+1}\right)}{\partial b^*_t}}}_{\text{Discipline effect}} \tag{72}$$

B.3 Calibrated model

Let $\mathbf{s}_t = \left(b^*_{t-1},\hat{b}_{t-1},\hat{b}^\pi_{t-1},y_{T,t}\right)$ denote the vector of state variables, where $\hat{b}_t = b_t/\alpha$ and $\hat{b}^\pi_t = b^\pi_t/\alpha$. The Lagrangian for the policy problem in the calibrated section is given by

$$\begin{aligned}
\mathcal{L} &= u(C(c_{T,t},y_N)) - l(\pi_t) + \beta E_t[V(s_{t+1})] \\
&+ \lambda_t \Bigg\{ y_{T,t} - c_{T,t} - b^*_{t-1} - \frac{c^{1-\alpha}_{T,t}}{\pi}\hat{b}_{t-1} - c^{1-\alpha}_{T,t}\hat{b}^\pi_{t-1} \\
&+ \frac{1}{R-\delta}(b^*_t-\delta b^*_{t-1}) + \frac{1}{R}E\left[\mathcal{X}(\mathbf{s}_{t+1})\right]\left(\hat{b}_t-\delta\frac{\hat{b}_{t-1}}{\pi_t}\right) + \frac{1}{R}E\left[\mathcal{Z}(\mathbf{s}_{t+1})\right]\left(\hat{b}^\pi_t-\delta\hat{b}^\pi_{t-1}\right) \\
&- \frac{\psi_b}{2}\left(\frac{1}{R-\delta}b^*_t-\bar{B}^*\right)^2 - \frac{\psi_b}{2}\left(\frac{1}{R}E\left[\mathcal{X}(\mathbf{s}_{t+1})\right]\hat{b}_t-\bar{B}\right)^2 - \frac{\psi_b}{2}\left(\frac{1}{R}E\left[\mathcal{Z}(\mathbf{s}_{t+1})\right]\hat{b}^\pi_t-\bar{B}^\pi\right)^2\Bigg\}
\end{aligned} \tag{73}$$

where the model equilibrium also involves the bond prices definitions given by

$$Q(s_t) = \frac{1}{c^{1-\alpha}_{T,t}}\frac{1}{R}E_t[\mathcal{X}(s_{t+1})] \tag{74}$$

$$Q^\pi(s_t) = \frac{1}{c^{1-\alpha}_{T,t}}\frac{1}{R}E_t[\mathcal{Z}(s_{t+1})] \tag{75}$$

and the process for tradable endowment (29).

The first order conditions of the problem are

$$c_T : \alpha c_T^{\alpha(1-\sigma)-1} = \lambda \left[1 + (1-\alpha) c_T^{-\alpha} \left(\frac{\hat{b}}{\pi} + \hat{b}^{\pi} \right) \right] \tag{76}$$

$$\pi : \psi(\pi - \bar{\pi}) = \lambda \left\{ \frac{c_T^{1-\alpha}}{\pi^2} \hat{b} + \frac{1}{R} E\left[\mathcal{X}(\mathbf{s}')\right] \frac{\delta \hat{b}}{\pi^2} \right\} \tag{77}$$

$$\begin{aligned}
b^{*\prime} : \beta E\left[V_{b^*}(\mathbf{s}')\right] = -\lambda \Bigg\{ & \frac{1}{R-\delta} + \frac{1}{R} E[\mathcal{X}_{b^*}(\mathbf{s}')] \left(\hat{b}' - \delta \frac{\hat{b}}{\pi} \right) + \frac{1}{R} E[\mathcal{Z}_{b^*}(\mathbf{s}')] \left(\hat{b}^{\pi\prime} - \delta \hat{b}^{\pi} \right) \\
& -\psi_b \left(\frac{1}{R-\delta} b^{*\prime} - \bar{B}^* \right) \frac{1}{R-\delta} - \psi_b \left(\frac{1}{R} E\left[\mathcal{X}(\mathbf{s}')\right] \hat{b}' - \bar{B} \right) \frac{1}{R} E\left[\mathcal{X}_{b^*}(\mathbf{s}')\right] \hat{b}' \\
& -\psi_b \left(\frac{1}{R} E\left[\mathcal{Z}(\mathbf{s}')\right] \hat{b}^{\pi\prime} - \bar{B}^{\pi} \right) \frac{1}{R} E\left[\mathcal{Z}_{b^*}(\mathbf{s}')\right] \hat{b}^{\pi\prime} \Bigg\}
\end{aligned} \tag{78}$$

$$\begin{aligned}
\hat{b}' : \beta E\left[V_b(\mathbf{s}')\right] = -\lambda \frac{1}{R} \Bigg\{ & E[\mathcal{X}_b(\mathbf{s}')] \left(\hat{b}' - \delta \frac{\hat{b}}{\pi} \right) + E[\mathcal{Z}_b(\mathbf{s}')] \left(\hat{b}^{\pi\prime} - \delta \hat{b}^{\pi} \right) + E[\mathcal{X}(\mathbf{s}')] \\
& -\psi_b \left(\frac{1}{R} E\left[\mathcal{X}(\mathbf{s}')\right) \hat{b}' - \bar{B} \right) \left(\frac{1}{R} E\left[\mathcal{X}_b(\mathbf{s}')\right] \hat{b}' + \frac{1}{R} E\left[\mathcal{X}(\mathbf{s}')\right] \right) \\
& -\psi_b \left(\frac{1}{R} E\left[\mathcal{Z}(\mathbf{s}')\right] \hat{b}^{\pi\prime} - \bar{B}^{\pi} \right) \frac{1}{R} E\left[\mathcal{Z}_b(\mathbf{s}')\right] \hat{b}^{\pi\prime} \Bigg\}
\end{aligned} \tag{79}$$

$$\begin{aligned}
\hat{b}^{\pi\prime} : \beta E\left[V_{b^{\pi}}(\mathbf{s}')\right] = -\lambda \frac{1}{R} \Bigg\{ & E[\mathcal{X}_{b^{\pi}}(\mathbf{s}')] \left(\hat{b}' - \delta \frac{\hat{b}}{\pi} \right) + E[\mathcal{Z}_{b^{\pi}}(\mathbf{s}')] \left(\hat{b}^{\pi\prime} - \delta \hat{b}^{\pi} \right) + E[\mathcal{Z}(\mathbf{s}')] \\
& -\psi_b \left(\frac{1}{R} E\left[\mathcal{X}(\mathbf{s}')\right] \hat{b}' - \bar{B} \right) \frac{1}{R} E\left[\mathcal{X}_{b^{\pi}}(\mathbf{s}')\right] \hat{b}' \\
& -\psi_b \left(\frac{1}{R} E\left[\mathcal{Z}(\mathbf{s}')\right] \hat{b}^{\pi\prime} - \bar{B}^{\pi} \right) \left(\frac{1}{R} E\left[\mathcal{Z}_{b^{\pi}}(\mathbf{s}')\right] \hat{b}^{\pi\prime} + \frac{1}{R} E\left[\mathcal{Z}(\mathbf{s}')\right] \right) \Bigg\}
\end{aligned} \tag{80}$$

Appendix C Solution method

This section describes the solution method used to solve the model described in section 3.

There are 4 state variables; three endogenous, b^*, $\hat{b} = \frac{1}{\alpha} b$ and $\hat{b}^{\pi} = \frac{1}{\alpha} b^{\pi}$; and one exogenous, y_T (assumption: $y_N = 1$). Let $\mathbf{s} = \left(b^*, \hat{b}, \hat{b}^{\pi}, y_T \right)$ denote the vector of state variables. There are 7 control variables: $c_T, \pi, b^{*\prime}, \hat{b}', \hat{b}^{\pi\prime}, Q, Q^{\pi}$. Correspondingly, there are 7 functional equations, defined by the set of FOCs, (77)-(80), the resource constraint (30), and the bond prices, (74) and (75), with

$$\begin{aligned}
\lambda &= \frac{\alpha c_T^{\alpha(1-\sigma)-1}}{1 + (1-\alpha) c_T^{-\alpha} \left(\frac{\hat{b}}{\pi} + \hat{b}^{\pi} \right)} \\
\mathcal{X}(\mathbf{s}') &\equiv \frac{{c'_T}^{1-\alpha}}{\pi'} \left(1 + \delta Q(\mathbf{s}') \right) \\
\mathcal{Z}(\mathbf{s}') &\equiv {c'_T}^{1-\alpha} \left(1 + \delta Q^{\pi}(\mathbf{s}') \right)
\end{aligned}$$

$$E[\mathcal{X}_{b^*}(\mathbf{s}')] = (1-\alpha)\frac{{c'_T}^{-\alpha}}{\pi'}\left(1+\delta Q(\mathbf{s}')\right)\frac{\partial c'}{\partial b^{*\prime}} - \frac{{c'_T}^{1-\alpha}}{(\pi')^2}\left(1+\delta Q(\mathbf{s}')\right)\frac{\partial \pi'}{\partial b^{*\prime}} + \delta\frac{{c'_T}^{1-\alpha}}{\pi'}\frac{\partial Q(s')}{\partial b^{*\prime}}$$

$$E[\mathcal{X}_{b}(\mathbf{s}')] = (1-\alpha)\frac{{c'_T}^{-\alpha}}{\pi'}\left(1+\delta Q(\mathbf{s}')\right)\frac{\partial c'}{\partial b'} - \frac{{c'_T}^{1-\alpha}}{(\pi')^2}\left(1+\delta Q(\mathbf{s}')\right)\frac{\partial \pi'}{\partial b'} + \delta\frac{{c'_T}^{1-\alpha}}{\pi'}\frac{\partial Q(s')}{\partial b'}$$

$$E[\mathcal{X}_{b^\pi}(\mathbf{s}')] = (1-\alpha)\frac{{c'_T}^{-\alpha}}{\pi'}\left(1+\delta Q(\mathbf{s}')\right)\frac{\partial c'}{\partial b^{\pi\prime}} - \frac{{c'_T}^{1-\alpha}}{(\pi')^2}\left(1+\delta Q(\mathbf{s}')\right)\frac{\partial \pi'}{\partial b^{\pi\prime}} + \delta\frac{{c'_T}^{1-\alpha}}{\pi'}\frac{\partial Q(s')}{\partial b^{\pi\prime}}$$

$$E[\mathcal{Z}_{b^*}(\mathbf{s}')] = (1-\alpha){c'_T}^{1-\alpha}\left(1+\delta Q^\pi(\mathbf{s}')\right)\frac{\partial c'}{\partial b^{*\prime}} + \delta {c'_T}^{1-\alpha}\frac{\partial Q^\pi(s')}{\partial b^{*\prime}}$$

$$E[\mathcal{Z}_{b}(\mathbf{s}')] = (1-\alpha){c'_T}^{1-\alpha}\left(1+\delta Q^\pi(\mathbf{s}')\right)\frac{\partial c'}{\partial b'} + \delta {c'_T}^{1-\alpha}\frac{\partial Q^\pi(s')}{\partial b'}$$

$$E[\mathcal{Z}_{b^\pi}(\mathbf{s}')] = (1-\alpha){c'_T}^{1-\alpha}\left(1+\delta Q^\pi(\mathbf{s}')\right)\frac{\partial c'}{\partial b^{\pi\prime}} + \delta {c'_T}^{1-\alpha}\frac{\partial Q^\pi(s')}{\partial b^{\pi\prime}}$$

I approximate a linear policy function for each control variable and, following the Taylor projection method developed in Levintal (2018), I set to zero all the functional equations and their partial derivatives with respect to each state variables. Thus, I have a system of 35 equations (7 functional equations and 28 partial derivatives) and 35 unknowns (constant plus 4 coefficients corresponding to each state variable, for each control variable). I first obtain a solution at one point of the grid using Ottonello and Perez (2019) as a benchmark. Then, I simulate the exogenous stochastic process for tradable endowment for 100,000 periods.

Appendix D Analysis of additional parameters

In addition to the parameters analyzed in subsection 5.2, I consider the effect of changing the risk aversion parameter, σ, and the share of tradables in aggregate consumption, α. Figure 3 plots these outcomes, considering the model calibrated to the average Latin American country as the baseline.

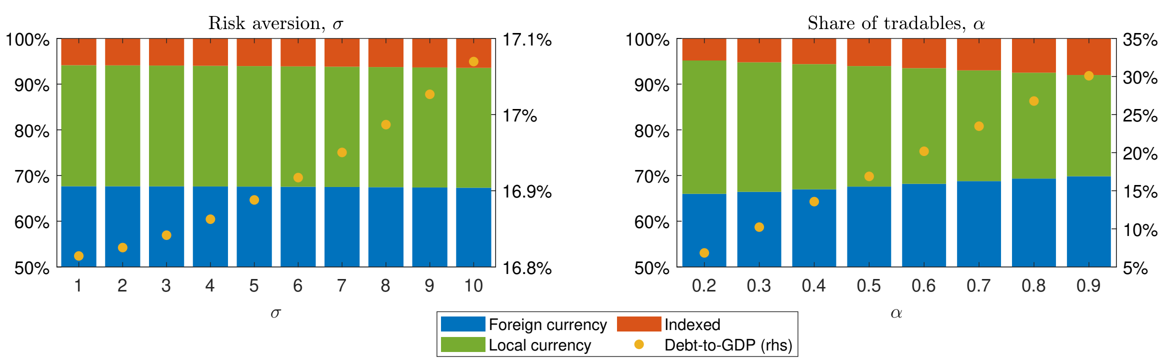

Figure 3: Currency composition (left-hand side axis) and average external debt-to-GDP (right-hand-side axis) for different parameter values. Except for the parameter subject to change, the calibration corresponds to the average Latin American country.

On the left panel of Figure 3 we observe that both currency composition and average external debt-to-GDP remain relatively stable for different levels of risk aversion, σ. This affects the marginal utility of consumption, therefore affects the hedging benefits (consumption smoothing), as well as the inflation and real exchange rate devaluation incentives. In this numerical results we observe that these benefits and costs get almost offset.

On the right panel of Figure 3 we observe that as α increases, there is a slight increase in the share of foreign currency and inflation-indexed debt, to the detriment of the share of local currency debt. Additionally, as α increases, so does the average external debt-to-GDP ratio. The share of tradables in aggregate consumption, α, affects the marginal utility of consumption as well as the real exchange rate, and therefore hedging benefits and costs derived from inflation and real exchange rate depreciation bias.